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Modelling the Volatility of Stock Markets

Modelování volatility akciových trhů

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1. Introduction
2. High Frequency Data in Finance
3. Volatility Modelling
4. Empirical Applications
5. Summary
6. Conclusions

Bibliography
List of Abbreviations
Declaration of Utilization of Results from the Diploma Thesis
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Annexes

References:

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TSAY, Ruey S. *Analysis of Financial Time Series.* 2nd ed. University of Chicago, New Jersey: John Wiley & Sons, 2005. ISBN 978-0-471-69074-0.

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The declaration

"Herewith I declare that I elaborated the entire thesis, including all annexes, independently."

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CONTENTS

1. Introduction

Recently years, financial model of time series take a significant role in plenty of financial industrial, even in decision making and economic forecast. One of the popular models is created for volatility, which has been developed by an extensive literature. At the same time, volatility model is widely applied in analyzing the characteristics of financial market, and testing some specific financial situations.

Especially in liquid stock market, volatility is considered a symptom. Generally increase stock market volatility brings a large stock price change of advances or declines. And then the raise of volatility is usually interpreted as an increase of risk in stock investment, which force investors shift their capital to less risky assts. Thus analysis of volatility in stock market makes some important sense, as our purpose of writing this diploma thesis. Absolutely, not only in stock market, volatility modeling is important for many issues, including leverage ratios, credit spreads, derivative pricing and portfolio decisions.

The traditional econometrics assumes that the volatility or variance of time series variables is constant, which is not practical. As an example, people recognized that the volatility of stock price return is not stable as time changing. It means that traditional econometrics cannot explain the fact well. In addition, according to the perspective of econometric inference, the loss in asymptotic efficiency from neglected heteroskedasticity may be extreme large. Therefore some of conditional heteroskedastic variance models are created for volatility measuring, such as ARCH GARCH, FIGARCH, IGARCH, EGARCH and so on.

In times of low market volatility, it is relatively easy to understand volatility dynamics and modeling volatility. But in some of times, volatility of financial time series is affected by infrequent market structural breaks, corresponding to economic turmoil. Examples include the 1997 Asian currency crisis, the IT dot com bubbles, and the recent global crisis in 2007 ~ 2010. During such periods, apparent spikes in volatility and large movements in asset prices complicate estimation and forecasting of volatility and understanding tis dynamics. These crises or political events dramatically influenced market volatility and diversification opportunities for investors.

The most conditional heteroskedastic variance models do not consider the shift or sudden break in volatility. However, there may be lots of potentially shock in volatility in fact, especially in emerging market. Thus, it is important to take account of these break events, when we estimate volatility persistence.

The main goal of this thesis is to identify sudden breaks in volatility and determine an impact of these breaks on the volatility of stock market. For the propose of this thesis, we use weekly time series of Chinese and U.S. stock markets, covering the period of 2000 ~2014 years. And then, the primary aim will be fulfilled by 3 steps.

In first stage, in order to detecting sudden breaks in both stock markets, we will utilize data samples with a help of the Iterated Cumulative Sums of Squares or ICSS algorithm. In this thesis, Chinese stock market is represented by Shenzhen Component Index, while American stock market is approximated by Dow Jones Industrial Average or DJIA Index.

Next step, we will evaluate an impact of sudden breaks on volatility persistence or long memory, by using General Autoregressive Conditional Heteroskedasticity (GARCH) mdoel and Fractional Integrated General Autoregressive Conditional Heteroskedasticity (FIGARCH) models.

Finally, there will be an in-sample test for fitting degree. And the forecasting ability would be presented as the performance of comparison between actual variances and expected variances, which is created by above-mentioned conditional heteroskedastic variance models.

This paper is divided into 6 chapters, and organised as follows:

The first chapter was the introduction of this thesis, and the main goal was presented as previous mentions.

Chapter 2 will describe some features and application of high frequency financial data. Firstly, there will introduce some characteristics of financial time series, including leptokurtic distribution, volatility clustering and leverage effect of volatility, which features will be observed in empirical application. And then, some important assumptions of financial time series are necessary to be mentioned and considered in our thesis, in order to construct conditional variance model. Finally, the microstructure of financial market will be presented.

Chapter 3 is about the methodologies of volatility model with and without shifts. Initially, we should know what kinds of conditional variance model will be used in this thesis. Afterwards, this chapter will provide the modeling estimation procedures and the criteria of model selection. Last stage, not only Iterative cumulative sum of squares or ICSS algorithm, which is the most significant method, but also models with dummy variables will be introduced.

Chapter 4 will provide the empirical application for achieving our goals. In this chapter, mentioned methods in Chapter 3 and important description of frequency financial data in Chapter 2 will be used. First and foremost, the investigated stock markets will be described. And then some analysis of statistical features will be presented about selected stock index. Furthermore, the main goal, identification of shock breaks and analysis of volatility persistence, will be discussed. In the final step, in-sample analysis will be present for testing the quality of GARCH(1,1) model with and without dummy variables.

Chapter 5 is the summaries for results of Chapter 4. All of the results of Chapter 4 will be listed and described specifically.

In the last chapter, there will be the conclusions of this thesis.

Ultimately, it must be claimed that all original data are received from public sources, and all of tables and figures are made by own calculations in EViews and R software.

2. High Frequency Data in Finance

High frequency data in finance are observations on financial variables taken daily or at a finer time scale. In addition, there is often irregularly interval over time. The excellent applications of high-frequency financial data sets in financial econometrics are provided by Andersen (2000), Campbell, Lo and MacKinlay (1997), Dacarogna et. al. (2001), Ghysels (2000), Goodhart and O'Hara (1997), Gouri´eroux and Jasiak (2001), Lyons (2001), Tsay (2001), and Wood (2000).

These high-frequency financial data sets have been widely used to study various market microstructure related issues, including price discovery, competition among related markets, strategic behavior of market participants, and modeling of real time market dynamics. Moreover, high-frequency data are also useful for studying the statistical properties, volatility in particular, of asset returns at lower frequencies by Ruey S (2005).

In this chapter, we will introduce some features and application of high frequency financial data.

2.1 Characteristic Features of Financial Time Series

In this subchapter, we will provide some characteristic features of financial time series, which include definition and some important parts like Leptokurtic distribution, volatility clustering and leverage effect, as defined by Jürgen and Christian M (2011).

In the first place, there are some descriptions of time series. A time series is that ordering and forming each numerical statistical indicator of a phenomenon according to time sequence, typically consisting of successive measurements made over a time interval. Time series method is a quantitative prediction method, also called simple extension method. Besides time series method is widely used as a common forecast method in statistics.

Furthermore, time series analysis is a kind of dynamic data processing method of statistics. The method is based on random process theory and mathematical statistical methods. At the most important point, time series analysis try to find some statistics law for selection random data sequence, then used for solving practical problems. If we use a model to predict future values based on previously observed values, this method can be call time series forecasting by Bauwens (2012).

When it comes to financial time series analysis, there are 2 key features for distinguishing financial time series analysis from other time series analysis.

- Financial time series analysis is more concerned with the theory and practice of asset valuation than others.
- Financial time series analysis is a highly empirical discipline, but like other scientific fields theory structures the foundation for making inference.

Last but not least, both financial theory and its empirical time series contain a few of uncertainty. For example, there are various definitions of asset volatility, and for a stock return series, the volatility is not directly observable. Due to a result of the added uncertainty, statistical theory and methods play an important role in financial time series analysis.

2.1.1 Leptokurtic Distribution

First and foremost, we must provide some descriptions of central moment measurement for understanding leptokurtic distribution. It is necessary to think about leptokurtic distribution, because most financial time series are not normal distribution and present a fat tails and excess kurtosis. And then let us look at this example:

The ξ th moment of a continuous random variable X is defined as:

$$E(X^{\xi}) = \int_{-\infty}^{+\infty} x^{\xi} f(x)dx \quad . \tag{2.1}$$

Where E stands for expectation and $f(x)$ is the probability density function of X. There are 4 measurements of central moment:

- The first central moment: *mean* $(\bar{x})$ or expectation of X. It measures the central location of the distribution.
- The second central moment: *variance* of X (σ_x^2) or *standard deviation* of X (σ_x). This central moment measures the variability of X.

The first two moments of a random variable uniquely determine a normal distribution. For determination of other distributions, we should consider third and fourth central moment.

- The third central moment: *skewness*. Commonly we use *skewness* to measure the symmetry of X with respect to its mean, as followed:

$$Skewness\ (x) = E\left[\frac{(X-\mu_x)^3}{\sigma_x^3}\right] \ . \tag{2.2}$$

- The fourth central moment: *kurtosis*. It used to measures the tail behavior of X usually, as followed:

$$Kurtosis\ (x) = E\left[\frac{(X-\mu_x)^4}{\sigma_x^4}\right] \ . \tag{2.3}$$

The amount of $K(x) - 3$ is called the *excess kurtosis* because *kurtosis* of normal distribution is 3. Thus, the *excess kurtosis* of a normal random variable is zero. According to the value of $K(x)$, we can find 2 types distribution with different unusual *kurtosis* shape:

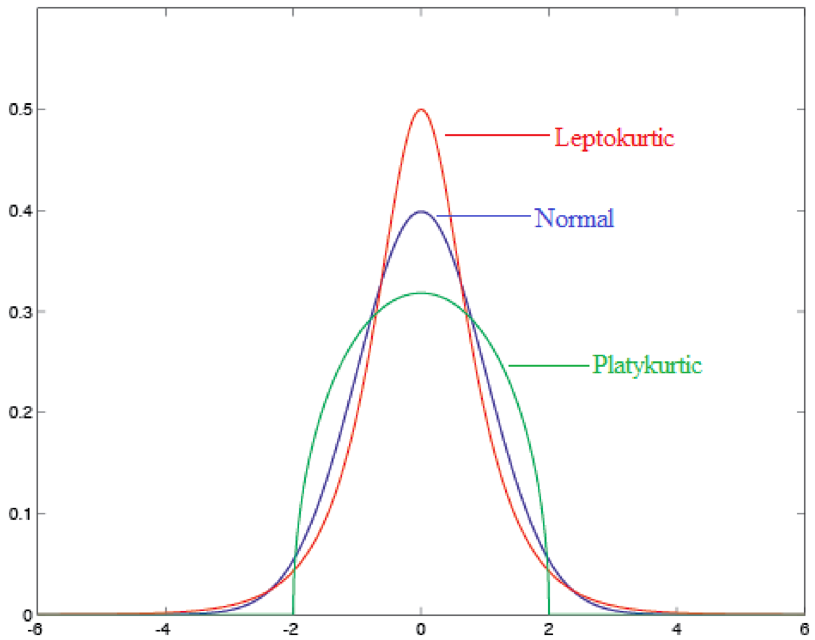
a) Leptokurtic distribution: A distribution with positive *excess kurtosis* would have heavy tails, implying that there are more mass of its support on the distribution tails than a normal distribution tails. In practice, this means that a random sample from such a distribution tends to contain more extreme values and mean value. For equity investment, leptokurtic distribution means most investors get profit in bull market and most investors experience loss in bear market. Besides fat tails imply volatility with long time-horizon.

b) Platykurtic distribution: A distribution with negative *excess kurtosis* has short tails (e.g., a uniform distribution over a finite interval). It means that there are less mass of its elements on the tails than a normal distribution.

The amount of $K(x) - 3$ is called the *excess kurtosis* because *kurtosis* of normal distribution is 3. Thus, the *excess kurtosis* of a normal random variable is zero. According to the value of $K(x)$, we can find 2 types distribution with different unusual *kurtosis* shape:

There is a clearly presentation about the difference between leptokurtic, normal and platykurtic distribution in Figure 2.1.

Figure 2.1: Distribution comparison among leptokurtic, normal and platykurtic



Source form: Raphaël Liégeois (2013) with own adjustment

2.1.2 Volatility Clustering

Market prices tend to exhibit periods of high and low volatility. This sort of behavior is called volatility clustering. In finance area, volatility clustering is observed by Mandelbrot (1963), which say "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes."

There is an example that situation of volatility clustering and non-clustering, which is presented in Figure 2.2, as followed:

Figure 2.2: Phenomenon of volatility clustering and non-clustering

Source form: http://www.riskglossary.com/link/volatility_clustering.htm

Moreover, there is a quantitative manifestation of this fact: while returns are uncorrelated each other, absolute returns $|r_t|$ or their squares display a positive, significant and slowly decaying autocorrelation function $\mathrm{corr}(|r_t|, |r_t + \tau|) > 0$ for τ ranging from a few minutes to several weeks, by Tsay, Ruey S (2005). It means that the slow decay behavior in autocorrelation functions of absolute returns is actually directly related to the degree of clustering of large fluctuations within the financial time series.

Since phenomenon of volatility clustering has intrigued many researchers and development in finance such as financial forecasting and derivatives pricing, GARCH models and stochastic volatility models are used primarily to model this phenomenon. In addition, the ARCH by Engle (1982) and GARCH by Bollerslev (1986) models aim to more accurately describe the phenomenon of volatility clustering and related effects such as kurtosis.

2.1.3 Leverage Effect of Volatility

One main theory that considers the relationship between volatility and equity price is the leverage effect by Black (1976) and Christie (1982). With the leverage effect, a negative return (declining price) increases financial leverage, making the stock riskier and increasing its volatility. The presence of the leverage effect would imply that the negative innovation (news) has a greater influence on volatility than a positive innovation (news).

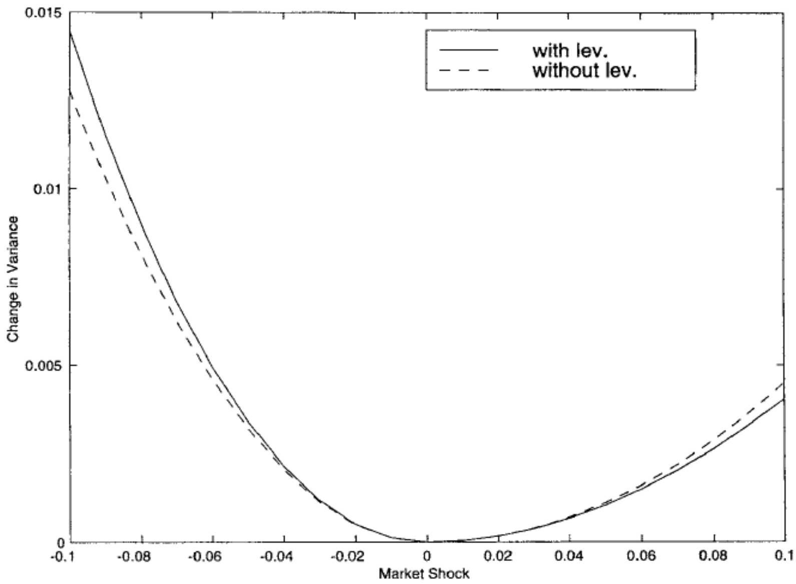
Since the leverage effect is a phrase that describes the asymmetric response of volatility to shocks of differing signs, ''leverage effects'' have become synonymous with asymmetric volatility. Black (1976) and Christie (1982) were among the first to document and explain a negative relationship between current individual stock return and future volatility in the US equity markets.

Black (1976) showed that if the price on day t fell then the volatility on day $t + 1$ would, on average, be higher than if the price rose by the same amount. Black's explanation of this phenomenon stated that a price fall would reduce the value of equity and hence increases the debt-to-equity ratio. Because this increase in leverage that raises the riskiness of the firm, we can observe an increase in volatility.

Christie (1982) tested Black's explanation by looking at the relationship between the asymmetry in equity volatility and the debt-to-equity ratio of firms. At the beginning he demonstrates that stock price changes and volatility are inversely related. For example, the elasticity of volatility with respect to the value of equity is negative. In addition, he also finds that volatility is an increasing function of financial leverage suggesting that this may be the cause of the negative elasticity of volatility with respect to the value of equity. He found a strong relationship between the leverage effect and the debt-to-equity ratio, but claimed that the debt-to-equity ratio cannot fully explain the effect. If such asymmetries volatility exist in individual stocks returns, it is natural to expect that in a cross sectional analysis the size of the asymmetry will be positively related to the degree of financial leverage. For example, the higher the leverage will lead to the more asymmetric the response of volatility to innovations.

Otherwise the asymmetric impact of innovations on volatility has to be explained by factors other than the financial leverage.

Figure 2.3: Asymmetric volatility



Source form: Geert Bekaert (2000) and Guojun Wu (2000)

This Figure 2.3 shows the market shock impact on market variance with or without incorporating the change in leverage level. The shocks are at the firm, not equity, level. Leverage ratios are evaluated at the sample mean except when leverage effects are taken into account.

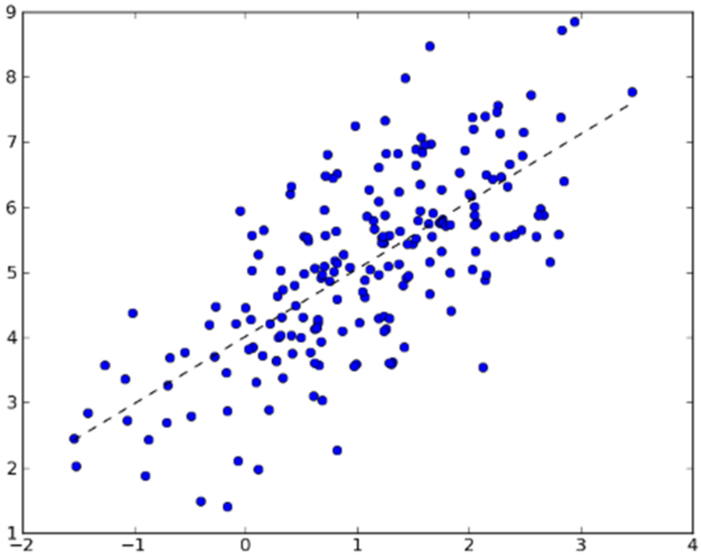
2.2 Assumptions of Financial Time Series

According to our mention in 2.1, we know that the financial time series is a set of financial observations for a variable over successive periods of time. Here we simply discuss the most application in financial time series analysis. That is linear regression analysis, also called ordinary least squares (OLS) regression analysis.

In statistics, ordinary least squares (OLS) or linear least squares is a method for estimating the unknown parameters in a linear regression model, with the goal of minimizing the differences between the observations in some arbitrary dataset and the responses of prediction by the linear approximation of the data (visually this is seen as the sum of the vertical distances between each data point in the set and the corresponding point on the regression line - the smaller the differences, the better the model fits the data).

There is a Figure 2.4, which presents an example of linear regression model by using ordinary least squares, as followed:

Figure 2.4: Linear regression model by ordinary least squares



Source from: http://mlpy.sourceforge.net/docs/3.3/lin_regr.html

For analyzing financial time series correctly, we must apply some assumption of linear regression. To ensure unbiased estimation and correct p-value, the main assumptions include:

- A linear relationship exists between the dependent and the independent variable.
- The independent variable is uncorrelated with the residuals and each other.
- The expected value of the residual term is zero. It means make a zero conditional mean assumption.

- The variance of the residual term is constant for all observations. It is known as homoscedasticity.
- The residual term is independently distributed; that is, the residual for one observation is not correlated with that of another observation. It is a statistical description as no autocorrelation.
- The residual term is normally distributed.

Then we divided previous 6 assumptions into 3 classifications, included normality, linearity and stationarity.

2.2.1 Linearity

That a linear relationship exists between the dependent and the independent variable is the assumption of linearity. In common application, linearity is referred to a mathematical relationship or function that can be graphically represented as a straight line, as in two quantities that are directly proportional to each other, such as the mass and weight of an object.

In financial time series, we need assume there is linearity between the object values (dependence) and time series (independence). This relationship can be presented as followed formula and Figure 2.5:

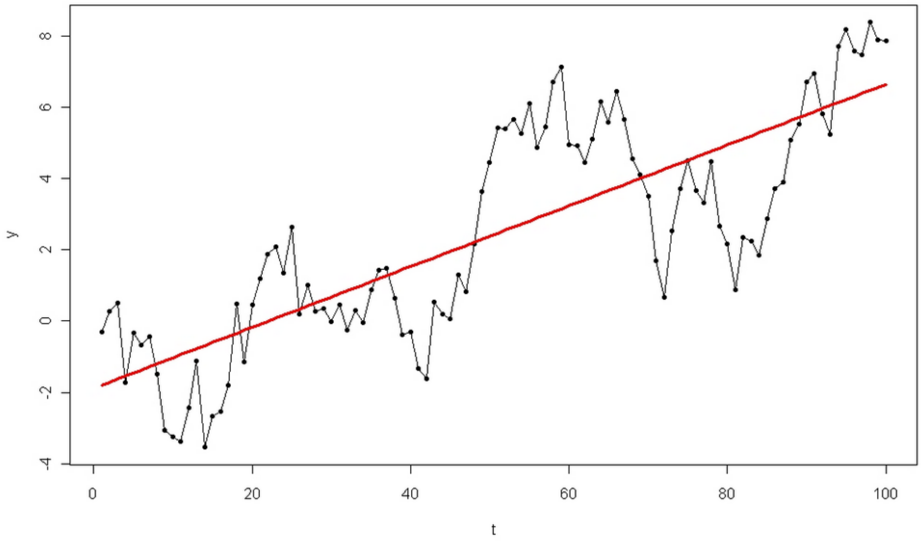
$$y_t = \beta_0 + \beta_1 \cdot t + \varepsilon_t \quad , \tag{2.4}$$

β_0: Coefficient of constant (intercept at the vertical axis),

β_1: Slope coefficient,

ε_t: Residual term.

Figure 2.5: Linearity time series model



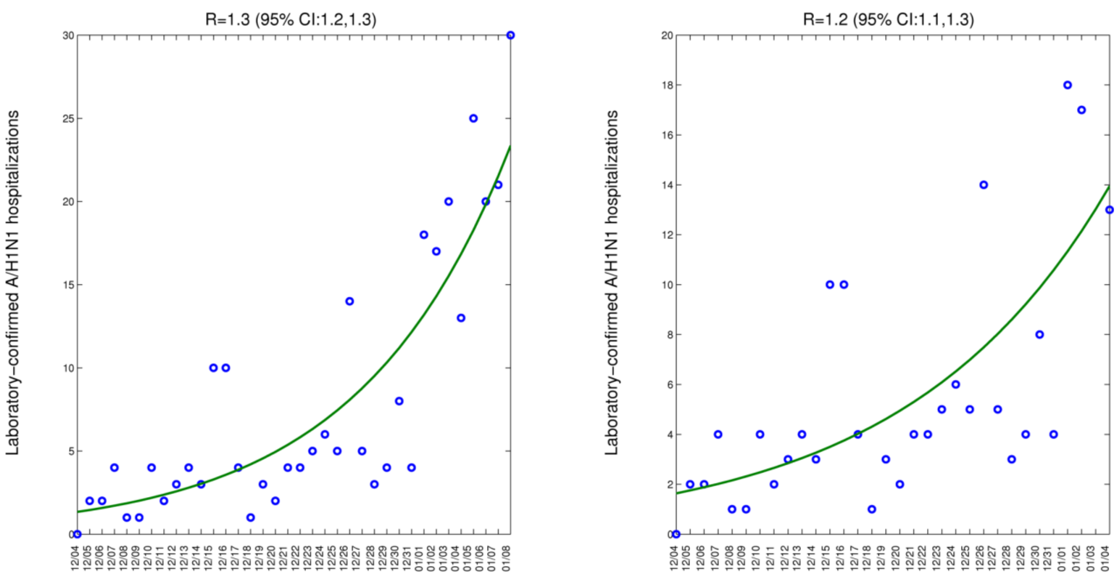
Source from: Rabett (2010)

Comparing with linearity time series model, we can present a non-linearity time series model as followed formula:

$$y_t = e^{\beta_0 + \beta_1 \cdot t} + \varepsilon_t \ . \tag{2.5}$$

and the Figure 2.6:

Figure 2.6: Non-linearity time series model



Source from: Catherine Mulbrandon (2010).

2.2.2 Normality

The next assumption can be called normality, which is important assumption for the autocorrelation test. In addition, normality is the precondition for construction of point forecasts and interpretation of the estimated parameters.

It means the residual term must follow normal distribution. The residual can be presented as:

$$\varepsilon_t = y_t - \hat{y}_t \ , \tag{2.5}$$

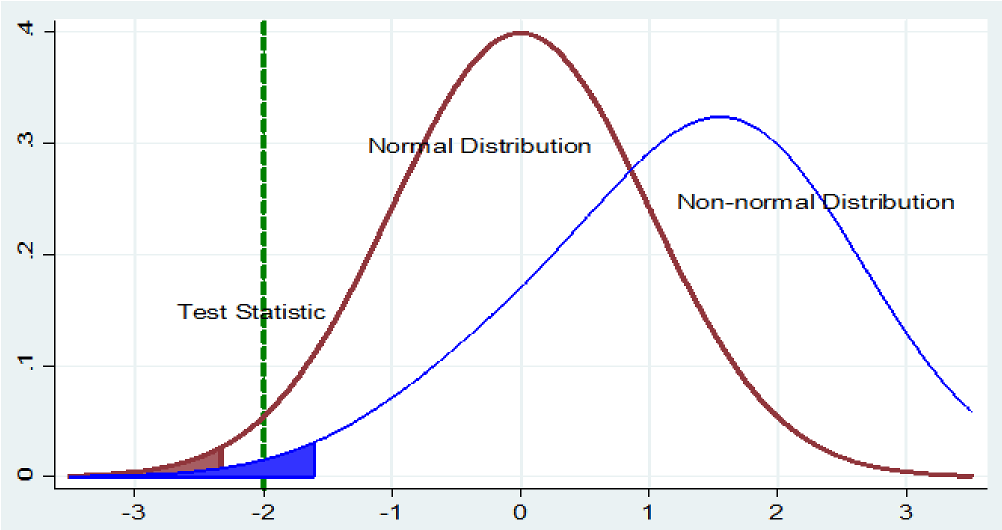
y_t: The value of the time series or the dependent variable at time t,

$\hat{y}_t$: The predicated value of y, which is the dependent variable at time t.

In probability theory, the normal or Gaussian distribution is a very commonly occurring continuous probability distribution—a function that tells the probability that any real observation will fall between any two real limits or real numbers, as the curve approaches zero on either side. Normal distributions are extremely important in statistics and are often used in the natural and social sciences for real-valued random variables whose distributions are not known.

We can simply compare the difference between normal distribution and non-normal distribution by the Figure 2.7.

Figure 2.7: Comparing between normal distribution and non-normal distribution



Source from: Hun Myoung Park (2009)

Besides, we must discuss the application of Jarque-Bera statistic, which is often used to test the null of whether the standardized residuals are normal distribution. Jarque-Bera test give us the result of probability of normally distributed instead of leptokurtic distribution, by testing skewness and kurtosis simultaneously.

The assumption of Jarque-Bera is that the third moment of skewness in normal distribution is zero and forth moment of kurtosis in normal distribution is 3. So the *JB* statistic is be defied as:

$$JB = (SK^2 + KU^2) \ , \tag{2.6}$$

where the *SK* is the test reiteria of skewness, it can be expressed as:

$$SK = (\frac{T}{6})^{\frac{1}{2}}(\frac{\hat{m}_3^2}{\hat{m}_2^3})^{\frac{1}{2}} \ , \tag{2.7}$$

and the *KU* of kurtosis test criteria is descripted as follows:

$$KU = (\frac{T}{24})^{\frac{1}{2}}(\frac{\hat{m}_4}{\hat{m}_2^2} - 3) \ , \tag{2.8}$$

while

$$\hat{m}_j = \frac{1}{T}\hat{\varepsilon}_t^j, \quad j = 2,3,4 \ . \tag{2.9}$$

Under the assumption of normality of unsystematic component, both statistics SK and KU have normal distribution $N(0,1)$. However JB statistic has the distribution of $\chi^2(2)$. In fact, not only non-normality of unsystematic component but also the heteroskedastic unsystematic component can result in refusing the null hypothesis.

2.2.3 Stationarity

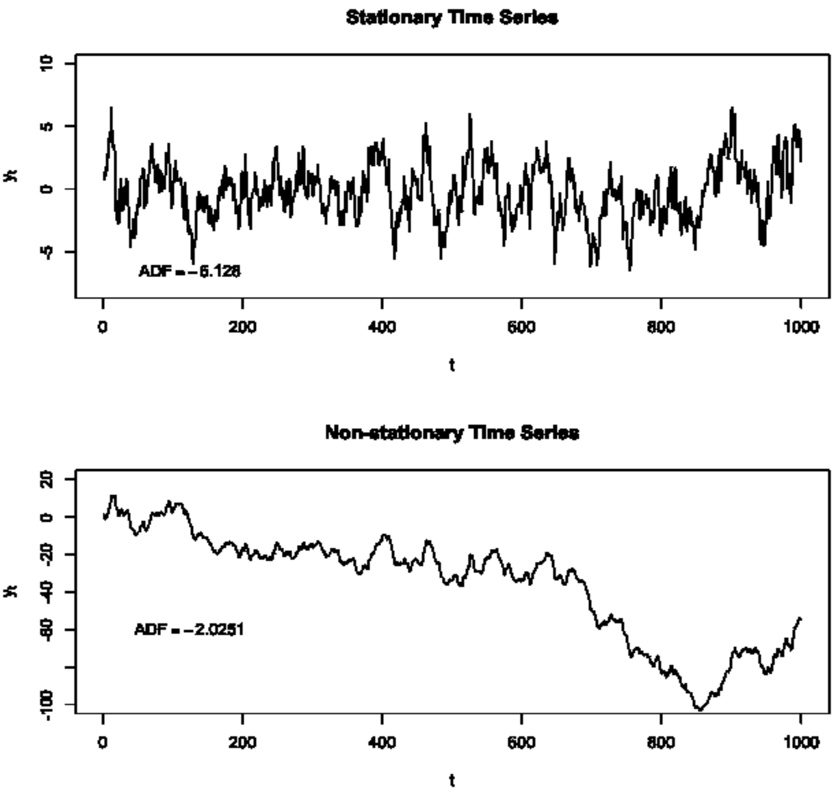
The rest 4 assumptions are about stationarity assumption.

In mathematics and statistics, a stationary process is a stochastic process whose joint probability distribution does not change as the shift of time. Ultimately, if parameters such as the mean and variance are presented, would also do not change over time and do not follow any trends.

Stationarity is used as a tool for time series analysis, where the raw data is often transformed to become stationary; for example, economic data are often seasonal and/or dependent on a non-stationary price level.

According to previous description, we can see some difference between stationarity and non-stationarity in Figure 2.7 as follows:

Figure 2.7: Comparing between stationarity and non-stationarity



Source from: *Nils Bohr*, *Thomson* and *Box*

The foundation of time series analysis is stationarity. A time series $\{r_t\}$ is said to be strictly stationary if the joint distribution of $(r_{t_1}, \ldots, r_{t_k})$ is identical to that of $(r_{t_1+t}, \ldots, r_{t_k+t})$ for all t, where k is an arbitrary positive integer and $(t_1, \ldots, t_k)$ is a collection of k positive integers. In other words, strict stationarity requires that the joint distribution of $(r_{t_1}, \ldots, r_{t_k})$ is invariant under time shift. This is a very strong condition that is hard to verify empirically.

A weaker version of stationarity is often assumed. A time series $\{r_t\}$ is weakly stationary if both the mean of r_t and the covariance between r_t and $r_{t-\ell}$ are time-invariant, where ℓ is an arbitrary integer. More specifically, $\{r_t\}$ is weakly stationary if:

a) $\mathrm{E}(r_t) = \mu$, which is a constant, and

b) $\mathrm{Cov}(r_t, r_{t-\ell}) = \gamma_\ell$, which only depends on ℓ. In practice, suppose that we have observed

T data points $\{r_t | t = 1, \dots, T\}$.

The weak stationarity implies that the time plot of the data would show that the T values fluctuate with constant variation around a fixed level. In applications, weak stationarity enables one to make inferences concerning future observations (e.g., prediction).

The covariance $\gamma_\ell = \mathrm{Cov}(r_t, r_{t-\ell})$ is called the lag-ℓ autocovariance of r_t. It has two important properties: (a) $\gamma_0 = \mathrm{Var}(r_t)$ and (b) $\gamma - \ell = \gamma_\ell$. The second property holds because $\mathrm{Cov}\left(r_t, r_{t-(-\ell)}\right) = \mathrm{Cov}\left(r_{t-(-\ell)}, r_t\right) = \mathrm{Cov}(r_{t+\ell}, r_t) = \mathrm{Cov}\left(r_{t_1}, r_{t_1-\ell}\right)$, where $t_1 = t + \ell$.

In the finance literature, it is usual to assume that an asset return series is weakly stationary. This assumption can be checked empirically provided that an enough number of historical returns are available.

2.3 Microstructure of Financial Markets

It is important to know microstructure of financial markets, when us try to price financial assets value and understand their internal design concept. There is a common definition of market microstructure, "It is the study of the trading mechanisms used for financial securities", which is defined by Maureen O'Hara (1995) of Cornell University. She is an authority on market microstructure, describes market microstructure as "the study of the process and outcomes of exchanging assets under a specific set of rules."

In addition, National Bureau of Economic Research (NBER) defines market microstructure as a field of study, which is devoted to theoretical, empirical, and experimental research on the economics of security markets. This study includes the role of information in the price discovery process, the definition, measurement and control of liquidity, and transaction costs and their implication for efficiency, welfare, and regulation of alternate trading mechanisms and market structures.

It seems that market microstructure is about designing a superior stock exchange at a superficial level. But market microstructure is much more than that. Market microstructure has broader interest among financial economists, since it has implications for asset pricing, international finance and corporate finance. A basic assumption of market microstructure theory is that asset prices need not reflect information expectations value fully due to a variety of frictions. Thus, market microstructure is related to the field of investments, which is care about the equilibrium value of financial assets. Market microstructure is also linked to traditional corporate finance, since difference between price and value has the potential to affect financing and capital structure decisions. The relationship between market microstructure and other areas of finance is relatively new and is continuing to evolve.

Furthermore market microstructure is a branch of finance concerned with the details of how exchange happens in financial markets. While the theory of market microstructure applies to the exchange of real or financial assets, more evidence is available on the microstructure of financial markets due to the availability of transactions data from them. The major thrust of market microstructure research examines the ways in which the working processes of a market affects determinants of transaction costs, prices, quotes, volume, and trading behavior. Recently innovations have allowed an expansion into the study of the impact of market microstructure on the incidence of market abuse, such as insider trading, market manipulation and broker-client conflict.

The most application of market microstructure can be summarize into 4 parts:

- Market structure and design issues: it focuses on the relationship between price determination and trading rules.
- Price formation and discovery: it concentrates on the process by which the price for an asset is determined.
- Transaction cost and timing cost: it pays attention to transaction cost and timing cost and the impact of transaction cost on investment returns and execution methods.
- Information and disclosure: it cares about the market information and transparency and the impact of the information on the behavior of the market participants.

3. Volatility Modeling

In recent years, volatility modeling became a popular studying in analyze the characteristics of financial market. And then an extensive literature has been developed for volatility analysis.

There is a traditional econometrics assumption that the volatility or variance of time series variables is constant, which is not practical. As an example, people recognized that the volatility of stock price return is not stable as time changing. It means that traditional econometrics cannot explain the fact well.

Understanding the accurate relationship between volatility and time, is crucial for many issues in macroeconomics and finance, such as irreversible investments, option pricing, the term structure of interest rates, and general dynamic asset pricing relationships. In addition, according to the perspective of econometric inference, the loss in asymptotic efficiency from neglected heteroskedasticity may be extreme large.

When evaluating economic forecasts, a much more accurate estimate of the forecast error uncertainty is generally available by conditioning on the current information set. It is the basic methodology of variance analysis.

In this chapter, we will describe some of volatility analysis models, which will be applied in empirical analysis of financial time series. There will be some of models, which can capture both long memory and nonlinear dynamics jointly in volatility process. They include ARCH model, GARCH model, FIGARCH model and the ICSS algorithm.

3.1 ARCH Model

For giving up this shortcoming and getting more accurate estimate of error, Autoregressive conditional heteroskedasticity model or ARCH model is presented by Robert F. Engle (1982). Engle obtained the 2003’s Nobel Prize of economy by ARCH model.

ARCH models are used to descript and model observations. They are used whenever there is reason to believe that, the error terms will have a characteristic size or variance at any point in a series. In particular ARCH models assume the variance of the current error term to be a function of the actual sizes of the previous time periods' error terms. Furthermore, the variance is often related to the squares of the previous error term.

ARCH models are used commonly in modeling financial time series for exhibiting time-varying volatility clustering. ARCH-type models are sometimes considered to be part of the family of stochastic volatility models, but strictly this is incorrect. Because we can get a completely pre-determined volatility at time t by given previous values.

For construction of ARCH model, firstly we need understand the equation of linear regression model, which is the workhorse of economic modeling. A univariate linear regression represents a proportionality relationship between two variables:

$$y_i = \alpha + \beta x + \varepsilon_i \ . \tag{3.1}$$

The preceding linear regression model presents that the expectation of the variable y is β times the expectation of the variable x plus a constant α. The proportionality relationship between y and x is not exact but subject to an error ε. In standard regression theory, the error ε is assumed to have a zero mean and a constant standard deviation σ. The standard deviation is the square root of the variance, which is the expectation of the squared error:

$$\sigma^2 = E(\varepsilon^2) \ . \tag{3.2}$$

It is a positive number that measures the size of the error. We call the assumption is homoscedasticity, when the expected size of the error is constant and does not depend on the size of the variable x. If the expected size of the error term is not constant, we call it as heteroskedasticity.

In describing ARCH/GARCH behavior, we focus on the error process. In particular, we assume that the errors are an innovation process, that is, we assume that the conditional mean of the errors is zero. We write the error process as:

$$\varepsilon_t = \sigma_t \cdot z_t \ , \tag{3.3}$$

where σ_t is the conditional standard deviation and the z terms are a sequence of independent, zero-mean, unit-variance, normally distributed variables. Under this assumption, the unconditional variance of the error process is the unconditional mean of the conditional variance. However the unconditional variance of the process variable does not coincide with the unconditional variance of the error terms in general.

Secondly, when we discuss financial and economic models, conditioning is often stated as regressions of the future values of the variables on the present and past values of the same variable. For example, if we assume that time is discrete, we can express conditioning as an autoregressive model:

$$x_{t+1} = \alpha_0 + \beta_0 x_t + \cdots\cdots + \beta_n x_{t-n} + \varepsilon_{t+1} \ . \tag{3.4}$$

The error term ε_i is conditional on the information I_i that, in this example, is represented by the present and the past *n* values of the variable x.

Besides the simplest autoregressive model is:

$$x_{t+1} = \alpha_0 + \beta_0 x_t + \varepsilon_{t+1} \ . \tag{3.5}$$

Thirdly, we can construct the simplest autoregressive conditional heteroskedasiticity model as Robert F. Engle invented in 1982:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + u_t \ , \tag{3.6}$$

where ε_t is the error term of x_t. In addition, u_t is the error term of ε_t. Furthermore this formula is called ARCH(1) model, since ε in *t* time is correlated with ε in only *t-1* time or one lagged value. Under an ARCH model, if the residual return is large in magnitude, there is a high probability that next period's conditional volatility σ_t^2 will be large in our forecast.

According to formula 3.2, we can improve the formula 3.6 in:

$$\sigma_t^2 = var(\varepsilon_t|\varepsilon_{t-1}) = E(\varepsilon_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \ . \tag{3.7}$$

In formula 3.7, there is no implication that ARCH(1) is non-stationary. It is just stating, that σ_t^2 and ε_{t-1}^2 are correlated. Due to $\sigma_t^2 \geq 0$, $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$ must to be ensured as positive variance. So we should consider the covariance stationary in 2 scenarios:

1. If $\alpha_1 = 0$, the conditional variance must equal with α_0. And then this time series is covariance stationary.

2. If $\alpha_0 = 0$, the conditional variance will change with fluctuation of ε_{t-1}^2. So this time series can not be said that meet the requirement of covariance stationary.

Moreover, when α_1 meets the condition of $0 < \alpha_1 < 1$, we can get the mean-reverting level according to mean reversion of ARCH(1) model by formula:

$$\sigma^2 = \varepsilon^2 = \frac{\alpha_0}{1- \alpha_1} \ . \tag{3.8}$$

According to this formula 3.8, we know the more α_1 close to 1, the more probability that this time series model is not covariance stationary or heteroskedasticity. On contrary, there is high probability to say it is covariance stationary or homoscedastic by Husek (2007).

Last but not least, since the ARCH(1) states that the conditional variance depends only on 1 lagged value, ARCH(1) is a simplest autoregressive conditional heteroskedasticity model and not practical. In fact, we should consider more lagged value, which have significant influence. Generally we consider that the ARCH(1) model is the ARCH(*q*) model' 1 order. So we can extend ARCH model to *q* order and express it as:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots\cdots + \alpha_q\varepsilon_{t-q}^2 + u_t \ . \tag{3.9}$$

Same as formula 3.7, we can get the expectation formula as:

$$\sigma_t^2 = var(\varepsilon_t) = E(\varepsilon_t^2) = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots\cdots + \alpha_q\varepsilon_{t-q}^2 \ . \tag{3.10}$$

As the result of formula 3.10, the conditional variance σ_t^2 depends on q lagged values. Only the effect of a shock happen between time q and time *t* that will be expressed by their coefficient α. But there will be no a influence for σ_t^2, if the periods older than q.

And then we can construct the mean-reverting level for ARCH(*q*):

$$\sigma^2 = \varepsilon^2 = \frac{\alpha_0}{1- \sum_{i=1}^{q}\alpha_i} \ . \tag{3.11}$$

Same as the analysis of covariance stationary in ARCH(1), we can get the same conclusion.

If $\alpha_i = 0$, we can conclude that the ARCH(q) model fulfill the conditions of covariance stationary. So the long term conditional variances σ_{t-i}^2 are consistent and equal to unconditional variance σ^2.

There are some shortcomings about the application of ARCH model in volatility analysis:

1. The first problem is the strict condition of non-negativity for all coefficients in the conditional variance equation.
2. And then the most important problem is the parameters estimation and significant test. As the amount of q orders increase, the restrict condition will be much more complicated. In most actual regression analysis, these conditions can not be fulfilled easily.
3. Last, there is also a problem to determine the length of delay squares residuals q. It means we should know how many order will be used in ARCH(q) model.

3.2 GARCH Model

In order to improve these shortcomings of ARCH(q), autoregressive conditional heteroskedasticity model were generalized by many ways. And the most popular way is generalized ARCH, or GARCH model by Bollerslev (1986) and Taylor (1986). The GARCH process is often preferred by financial modeling professionals. It provides a more practical context than other forms, when trying to predict the prices and rates of financial instruments.

This model is also a weighted average of past squared residuals, but it has declining weights that never go completely to zero. In its most general form, it is not a Markovian model, as all past errors contribute to forecast volatility. It gives parsimonious models that are easy to estimate and, even in its simplest form, has proven surprisingly successful in predicting conditional variances especially in case of small selections. The GARCH model specified the conditional variance to be a function of lagged squared errors and past conditional variance, which replace the infinite length of delay q and expectation parameters in ARCH(q) model.

The most widely used GARCH specification asserts that the best predictor of the variance in the next period is a weighted average of the long-run average variance, the variance predicted for this period, and the new information in this period that is captured by the most recent squared residual. Such an updating rule is a simple description of adaptive or learning behavior and can be thought of as Bayesian updating.

Firstly we should understand the simplest GARCH model, which can be presented as:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + u_t \ , \tag{3.12}$$

where the u_t is the white noise followed random distribution in time *t* and defined as $\varepsilon_t^2 - \sigma_t^2$. In addition, the σ_{t-1}^2 is the past conditional variance, which is most use the weighted average of the long-run average variance.

And then if we improve the formula 3.12 as we do in ARCH model, we can get:

$$\sigma_t^2 = var(\varepsilon_t | \varepsilon_{t-1}) = E(\varepsilon_{\mathrm{t}}^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \ . \tag{3.13}$$

GARCH model described in formula 3.13 and typically referred to as the GARCH(1,1) model, which is widely used. This name derives from the fact that the 1,1 in parentheses is a standard notation, in which the first number refers to the number of autoregressive lags or ARCH terms and the second number refers to the number of moving average lags specified or often called the number of GARCH terms.

GARCH(1,1) model is better to explain volatility clusters than ARCH model. And then there is a same requirement for ensure non-negativity conditional variance σ_t^2, which means all these three parameters must be non-negativity, like $\alpha_0 \geq 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$.

Furthermore similarly to ARCH model, if α_1 and β_1 are between 0 and 1, we can also get the mean-reverting level by:

$$\sigma^2 = \varepsilon^2 = \frac{\alpha_0}{1-(\alpha_1+\beta_1)} \ . \tag{3.14}$$

When this GARCH(1,1) model is covariance stationary, the value of $\alpha_1 + \beta_1$ must be close to 0. Absolutely if the value of $\alpha_1 + \beta_1$ is close to 1, GARCH(1,1) is not covariance stationary. In addition, when we analyze the long memories effect of market shock, we can conclude that the more close to 1, the more persistence influence of shock will be.

Secondly if we expend GARCH(1,1) model to GARCH(p,q) model, it can be processed as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \ , \tag{3.15}$$

where the p is the length of delay $\varepsilon_t^2$and q represents the maximal length of delay σ_t^2. Besides the conditional variance should meet the requirement of non-negativity, which means $\alpha_0 \geq 0,\ \alpha_i \geq 0$ and $\beta_j \geq 0$.

Same as the GARCH(1,1), when $(\alpha_1 + \beta_1) - (\alpha_2 + \beta_2) - \cdots - (\alpha_m + \beta_m) \leq 1$ is fulfilled, there is also a mean-reverting level in GARCH(p,q):

$$\sigma^2 = \frac{\alpha_0}{1-[\,(\alpha_1+\beta_1)-(\alpha_2+\beta_2)-\cdots\cdots-(\alpha_m+\beta_m)]} \ . \tag{3.16}$$

Even though the GARCH(p,q) is widely useful and popular way in construction of model, there are still some limitations for application by Husek (2007):

1. In practical application, some of the estimated parameters can not meet the requirement of non-negativity.
2. The GARCH models are not able to explain the leverage effect, which means the asymmetric volatility mentioned in chapter 2.
3. It is hard to get the feedback between conditional variance and conditional average of regression model directly.

3.3 FIGARCH Model

In this subchapter, we consider another modification of ARCH family models to avoid some shortcoming of ARCH model. This model is Fractional Integrated GARCH morel or FIGARCH model by Baillie, Bollerslev, and Mikkelsen (1996).

The importance of introducing FIGARCH model was, this model provided a greater flexibility process for modeling the conditional variance. In addition, FIGARCH model is able to explain and represent the observed temporal dependencies in financial market volatility. Particularly the FIGARCH model allows only a slow hyperbolic rate of decay for the lagged squared or absolute innovations in the conditional variance function. What is more, this model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with some structure shifts on long memory of volatility.

By Baillie, Bollerslev, and Mikkelsen (1996) description, we can improve the GARCH(1,1) model and define the FIGARCH(1,d,1) model for a conditional variance as:

$$\sigma_t^2 = \alpha_0(1-\beta_1)^{-1} + [1-(1-\beta_1 L)^{-1}(1-\alpha_1 L)^{-1}(1-L)^d]\varepsilon_t^2 \ , \qquad (3.17)$$

where d is the fractional difference parameter, which fulfill the condition of $0 \leq d \leq 1$. Besides L means the lag operator, and $(1-L)^d$ present the first difference operator with the fractional difference parameter d.

When we use FIGARCH model to estimate volatility of persistence shocks, either the conditional variance or the level of long memory will be measured by the fractional differencing parameter d, which is estimated by quasi-maximum likelihood estimation technique.

3.4 Modeling Estimation Procedures

As we mentioned in previous sub-chapter, we should also develop an idea to establish the procedures of volatility models. Generally, the procedures can be divided into the following steps:

1. Determine a suitable linear or nonlinear model, based on the specified time series.
2. Test the hypothesis of conditional homoskedasticity against the alternative hypothesis of conditional heteroskedasticity for linear and nonlinear type.

3. Estimate the parameters of selected model for conditional heteroskedasticity.

4. Make a verification of suitability in the given model.

5. Make some modification for the model, if necessary.

6. Use the model for descriptive or predictive purposes.

3.5 Criteria for the Model Selection

There can be more than one acceptable estimated model, when analyzing the same time series. There are some methods available to choose the optimal one. The idea for choosing the optimal estimated model is to compare the residuals of each estimated model by the summary statistics.

Firstly, the best suitable estimation model can be chosen by the order of differentiation. The criteria are Akaike or AIC and Schwartz-Bayes or SBC by Arlt (2003).

Akaike information criterion can be expressed as:

$$AIC(M) = T\, ln\, \hat{\sigma}_{\varepsilon}^{2} + 2M \quad , \tag{3.18}$$

where the $M = p + q$ is the number of parameters in ARMA(p,q) model and $\hat{\sigma}_{\varepsilon}^{2}$ is the residual variance of this model. The result of minimum value of this criterion should be chosen.

And then, the Schwartz-Bayes criteria can be described as followed:

$$SBC(M) = T\, ln\, \hat{\sigma}_{\varepsilon}^{2} + M\, ln\, T \quad . \tag{3.19}$$

The difference between formula 3.17 and formula 3.18 is that in equation formula 3.18, T represents the number of observations. This T is equal to the number of residuals obtained from the model. The choose principal of Schwartz-Bayes criteria is same as formula 3.18, which means that the model with the minimum value of statistic will be optimal.

Secondly, in order to verify whether the model we have established is accurate enough for prediction, in other ward whether the estimation model can explain the real situation well, we can also use loss functions to make estimation about the size of differences between the theoretical result and actual result.

Root mean square error or RMSE is frequently used for measuring for differences between estimated data and the actual observed data. Basically, the RMSE represents the sample standard deviation of the differences between predicted values and observed values. Formula of RMSE is expressed as:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}(\hat{\sigma}_t - \sigma_t)^2}{n}} \ . \tag{3.20}$$

Besides, there are also other ways to measure the difference between estimation data and actual data, such as Mean Absolute Errors or MAE, Mean Absolute Percentage Errors or MAPE, Theil's Coefficient and so on. These criteria can be defined as follow:

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\hat{\sigma}_t - \sigma_t| \ , \tag{3.21}$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\frac{|\hat{\sigma}_t - \sigma_t|}{\sigma_t} \ , \tag{3.22}$$

$$Theil's\ U = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}|\hat{\sigma}_t - \sigma_t|}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}{\sigma_t}^2} + \sqrt{\frac{1}{T}\sum_{t=1}^{T}{\hat{\sigma}_t}^2}} \ . \tag{3.23}$$

The choice of a loss function is not arbitrary. It is very restrictive and sometimes loss function may be characterized by its desirable properties. In general, the loss function with lower value is the better estimation model, which has higher fitting degree.

Sound economical prediction practice requires selecting an estimator consistent with the actual acceptable variation experienced in the context of a particular applied problem. As a result, in the applied use of loss functions, we should be careful when deciding which statistical method to use to model an applied problem.

3.6 Iterative Cumulative Sum of Squares (ICSS) and Modification

In this subchapter, we will discuss the Iterative cumulative sum of squares or ICSS algorithm by Inclán and Tiao (1994) and its modification by Sanso (2004), because this quantitative method is an important algorithm in volatility analysis. Absolutely it will be used in chapter 4 about practical application.

Volatility of stock returns is affected substantially by infrequent sudden changes or regime shifts, corresponding to domestic and global economic events, such as financial crisis.

There will be some systematic effect on the whole market or might only affect a particular sector, when sudden events happen. For achieve efficient portfolios, the most of individual investors prefers to hold mutual funds or sector index funds rather than holding individual securities with some sudden event risk.

Furthermore, the investors and managers of index funds need to determine whether these major events cause shifts in volatility in the whole market or a particular sector, in order to create much better diversified portfolios, predict the future volatility of some index funds properly and get the accurate valuation of them.

However, if we only use the GARCH and FIGARCH approach to analyze the volatility, it is not enough since that GARCH and FIGARCH model are incapable of responding sudden changes. In addition, they are also inappropriate for examinations of volatility persistence.

In order to overcome this problem, the most popular approach is cumulative sums of squares (ICSS) algorithms, which was be stated by Inclán and Tiao (1994). ICSS algorithm is implemented to detect break point in the variance of a stock return series each sub-period.

There is the assumption of ICSS algorithm, which can be divided into 4 paths:

- Firstly, financial time series displays a stationary variance over an initial time period.
- And then there is a sudden shock that alters the variance when the certain event generates a break point.
- Moreover the variance will return to stationary again until next shock hits the market.

- This process is repeating over the time.

Firstly we should focus on formula 3.4, which is the autoregressive model presented previous subchapter, when we start to explain the ICSS algorithm.

And then Let us assume that $\{\varepsilon_t\}$ denote an independent time series with a zero mean and an unconditional variance, σ_j^2. In addition, the variances of each interval are given by $j = 0,1,\cdots,N_T$, where N_T is the total number of variance changes in T observations and $1 < K_1 < K_2 < \cdots < K_{N_T} < T$ are the set of change points.

To summarize, the unconditional variance σ_j^2 over N_T intervals can be defined as follow:

$$\sigma_t^2 = \begin{cases} \sigma_0^2,\ 1 < t < K_1 \\ \sigma_1^2,\ K_1 < t < K_2 \\ \vdots \\ \sigma_{N_T}^2,\ K_{N_T} < t < T \end{cases} . \tag{3.24}$$

Secondly a cumulative sum of squares is utilized to for determining the total number of sudden changes in volatility and the time point when each variance break or shift occurs. As the result, the cumulative sum of squares from the first observation to the K^{th} time point can be expressed as below:

$$C_K = \sum_{t=1}^{K} \varepsilon_t^2 \ , \tag{3.25}$$

where $K = 0,1,\cdots,T$.

Thirdly ICSS algorithm is based on D_K statistics and tests the null hypothesis of constant unconditional variance. And then D_K statistics is computed as follows:

$$D_K = \left(\frac{C_K}{C_T}\right) - \frac{K}{T} \quad , \tag{3.26}$$

where $D_0 = D_T = 0$. In which C_T is the sum of the squared residuals from the whole sample period. And then we can discuss 2 scenarios by variance stationary analysis as:

a) If there is no changes in variance occur, the D_K statistic will oscillate around zero. In addition, if D_K is plotted against K, it will resemble a horizontal line.

b) On the other hand, if there are one or more changes in variance occur, then the statistic values are not stable. D_K will be drift up or down from zero.

In this context, structure changes in variance are detected by the critical values obtained from the distribution of D_K assuming the null hypothesis of constant variance. Therefore, If the maximum absolute value of D_K is greater than the critical value, the null hypothesis of homogeneity can be rejected.

Last, the test proposed by Inclán and Tiao (1994) can be written as follows:

$$IT = sup_K\left|\sqrt{T/2}D_K\right| \ , \tag{3.27}$$

where $\sqrt{T/2}$ is used to standardise the distribution. And then let us define $K*$ as a value, which is the point of K when $sup_K|D_K|$ is obtained. If $sup_K\left|\sqrt{T/2}D_K\right|$ exceeds the critical value, then the parameter $K*$ means the time point when a volatility suddenly changed.

In accordance with the study of Inclán and Tiao (1994), the critical value of 1.358 is the 95th percentile of $sup_K\left|\sqrt{T/2}D_K\right|$ assuming that follow asymptotic distribution. Therefore, the upper and lower boundaries can be established at ± 1.358 in the D_K plot. So that a change point in variance is identified, when the value of D_K exceeds these boundaries.

The ICSS algorithm works by evaluating the D_K function over different time periods, and those different periods are determined by break points, which are themselves identified by the D_K plot. This algorithm was widely evidenced that financial data have time-varying variance and excess kurtosis

However, if the series harbors multiple change points, the D_K function alone will not be sufficiently powerful to detect the change points at different intervals. In this regard, Inclán and Tiao (1994) modified an algorithm that employs the D_K function to search systematically for change points at different points in the series.

Specifically, recent literature has shown that the ICSS algorithm tends to overstate the number of actual structural breaks in variance. Especially in the behavior of the ICSS algorithm is questionable under the presence of conditional heteroskedasticity, which is point out by Bacmann and Dubois (2002). They show that one way to circumvent this problem is by filtering the return series by a GARCH(1,1) model, and applying the ICSS algorithm to the standardized residuals. Bacmann and Dubois conclude that structural breaks in unconditional variance are less frequent than it was shown previously.

In order to overcome the problem that the size distortions are more extreme for heteroskedastic conditional variance processes, which make ICSS test invalidate in empirical application of financial time series. There are 2 modifications, which are proposed by *Sanso* et al (2004).

The first statistic k_1 is defined as follow:

$$k_1 = sup_K \left| \frac{1}{\sqrt{T}} B_K \right| \ , \tag{3.28}$$

where

$$B_K = \frac{C_K - \frac{K}{T} C_T}{\sqrt{\hat{\eta}_4 - \hat{\sigma}^4}} \ , \tag{3.29}$$

$$\hat{\eta}_4 = \frac{1}{T} \sum_{i=1}^{T} \varepsilon_t^4 \ , \tag{3.30}$$

and

$$\hat{\sigma}^2 = \frac{1}{T} C_T \ . \tag{3.31}$$

The second statistic k_2 is defined as follow:

$$k_2 = sup_K \left| \frac{1}{\sqrt{T}} G_K \right| \ , \tag{3.32}$$

where assuming that

$$G_K = \frac{1}{\sqrt{\hat{\omega}_4}} \left(C_K - \frac{K}{T} C_T \right) \ . \tag{3.33}$$

For estimating $\widehat{\omega}_4$ consistently, there are two possible ways given by *Sanso* et al (2004). The one ways is a nonparametric estimator, which is defined as:

$$\widehat{\omega}_4 = \frac{1}{T}\sum_{t=1}^{T}(\varepsilon_t^2 - \hat{\sigma}^2) + \frac{2}{T}\sum_{t=1}^{m} w(l,m)\sum_{t=l+1}^{T}(\varepsilon_t^2 - \hat{\sigma}^2)(\varepsilon_{t-l}^2 - \hat{\sigma}^2) \ . \qquad (3.34)$$

And another ways is parametric estimator, which can be described as:

$$\widetilde{\omega}_4 = (1 - \sum_{j=1}^{p}\lambda_j)^{-2}T^{-1}\sum_{t=1}^{T} e_t^2 \ , \qquad (3.35)$$

where λ_j and e_t are from the regression:

$$u_t = \hat{\delta} + \sum_{j=1}^{p}\hat{\lambda}_j u_{t-j} + e_t \ , \qquad (3.36)$$

with $u_t = \varepsilon_t^2 - \hat{\sigma}^2$.

Using the modified ICSS algorithm denotes that we are calculating test statistics for various sample sizes. Thus, the critical values for each test statistic might be obtained from a response surface as provided by Sanso et al (2004), as long they provide better results when using small samples.

Last, if we consider that sudden changes in volatility, the principal objectives of this study are twofold with a help of the ICSS algorithm:

1. This study detects the sudden changes using the ICSS algorithm, which is already introduced previously.

2. We examines whether the inclusion of sudden breaks in the GARCH model and FIGARCH model reduces the coefficients of volatility asymmetry and persistence or not.

Therefore, conditional variance in the GARCH(1,1) with sudden changes can be defined via the ICSS algorithm as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + d_1 D_1 + \cdots + d_n D_n \quad . \tag{3.37}$$

And then the FIGARCH(1,d,1) models with sudden changes will be described via the ICSS algorithm as:

$$\sigma_t^2 = \alpha_0 (1-\beta_1)^{-1} + [1 - (1-\beta_1 L)^{-1}(1-\alpha_1 L)^{-1}(1-L)^d]\varepsilon_t^2$$

$$+ d_1 D_1 + \cdots + d_n D_n \quad , \tag{3.38}$$

where $D_1 + \cdots + D_n$ denote dummy variables that can be equal the value of 1 when sudden shifts in volatility appears, elsewhere take a value of 0.

4. Empirical Applications

The previous Chapter 3 of this thesis was focused on the theoretical methodology. In this chapter, we will combine the mentioned methods and the real financial time series. In addition, the main purpose of this chapter is same as the aim of this thesis. We will examine the impact of structural breaks on volatility persistence of not only developed economies represented by American stock market, but also emerging economics represented by Chinese stock market.

The conducted steps in this chapter will be summarized as follow:

In Subchapter 1, the investigated stock markets will be described.

In Subchapter 2, some analysis of statistical features will be presented about selected stock index.

In Subchapter 3, the shock breaks will be identified respectively for returns of selected stock index.

In Subchapter 4, estimation models of conditional volatility will be created, as mention in Chapter 3. And the effect of volatility persistence or long memory will be analyzed.

In the last subchapter, in-sample analysis will be present for testing the quality of GARCH(1,1) model with and without dummy variables.

4.1 Description of Investigated Stock Markets

It is important to understand some information about the selected market, when we make a study for financial time series data. As the more information we know in a market, we can find the more structural break events easily.

In this subchapter, we will introduce the brief information about Chinese stock market and American stock market.

4.1.1 Features of Chinese Stock Market

There was a change in China, from centrally planned economy in 1950s ~ 1960s to more market orientated economy after 1980s. Currently, China is one of most significant participants in the global economy. And then Chinese emerging stock market also experienced a tremendous growth and development.

There are two Stock Exchanges in China, Shanghai Stock Exchange and Shenzhen Stock Exchange, which were established in 11/26/1990 and 12/01/1990 respectively. Generally, Shanghai Exchange is dominated by larger-cap companies, whereas in Shenzhen small joint-ventures and export-oriented company are listed. In addition, both exchanges are supervised by Chinese Securities Regulatory Committee or CRSC, which supervises new stock listing and daily trading activities.

In fact, it is proved that number of listed companies at both of Stock Exchanges grew up more than 100 times since 1990, and total market capitalization is far more than 500 billion USD. However, there was not a boom development in Chinese equity market before 2005. The reasons could be different, but the one main point should be, that Chinese currency was pegged to USD. When the foreign exchange regime was little liberalized, Chinese stock market started to experience a boom. A good example of this is that Shanghai Composite Index went up almost 130% and Shenzhen Composite Index even more as 197% in 2006.

Despite there is an amazing growth and development in Chinese equity market over the period of 20 years since 1990, the growth has been uneven and irregular. Especially in 2007~2008 global financial crisis, there is no denying that Chinese stock market experienced a dramatic decline. Therefore, we can conclude that Chinese stock market still remains in rather early stage of development, and it is necessary to reform and develop this inefficient market.

Furthermore, Chinese stock market has some unique, idiosyncratic features, which can be viewed in 6 aspects:

1. There are plenty of different classes of shares in China.

Chinese stock market actually consists of several sub-markets, with limited access and ability to buy stock and shares. Some of equity has limited access to either foreign or domestic investors, such as the restriction between A and B class.

To be more specific, On the one hand investments of A class shares are restricted only to domestic investors with rare exceptions. A class shares go public either on Shanghai or Shenzhen exchange, and they are denominated in not freely exchangeable RMB. On the other hand, foreign investors can trade B class shares, but these shares don't carry ownership rights in a company.

As we said about the sub-market in Chinese equity market, we have to mention H, N, L or S class shares, listed respectively in Honk Kong, New York, London and Singapore. What is more, most China's blue-chips are listed only on overseas exchanges, restricted only to foreign investors. This situation generates some structural drawbacks of Chinese stock market.

2. China market lacks true blue-chip stocks with high level of profitability. Besides it is dominant that the size gap between Chinese stock and others country's.

There are very few stocks that would fit the definition of "blue-chip" trading on China's mainland exchanges. While most developed markets are dominated by a limited number of large-cap stocks, China's market is cramped by a multitude of small-cap stocks. Chinese companies are not operating on high level of profitability, instead of providing rather poor earnings, and low dividend yields. This feature allows for increased speculation and higher turnover for both investors and indexes, among other problems.

3. Chinese equity market is dominated by retail investors.

Holdings of institutions in China are not more than 20% of total market share, comparing to 40% in U.S.A, U.K or Japan. Moreover, holdings of foreign investors in China are not exceeding 2.5%, while in developed markets of U.S.A and Japan they own respectively 11% and 19%.

4. Expansion through the issuance of new shares is rather than the appreciation in value of existing stocks.

Since these existing shares generally do not experience sustained growth, often because of market manipulation, they tend to be smaller size stocks in China's market. As the result, the current structure is a major obstacle to the creation of viable index-related products in China.

5. To be honest, there is a serious speculation situation in Chinese equity market.

Speculation is a very widespread phenomenon throughout China's stock market in different kind of investors. Most of individual investors are fighting for financial survival in a risky investing environment, also institutional investors frequently engage in speculation. Furthermore, Chinese stock market is not really encouraging long-term investment strategies with stable yield, instead short term speculation strategies are use more.

At the same time, speculation in Chinese stock market provides liquidity and encourages trading activity. For example, the excessive stock market trading can be observed almost every trading day.

6. It is a phenomenon that the stock market is yet to be regulated strictly.

Chinese government strictly regulates initial public offerings. Normally it is desirable to have supervision over fairness of operations, but in China it is even more complicated. Size, allocation and development of stock market in china are fully under control of state. The government controls the quota for new listings that can be issued each year, even selects qualified issuers based on sector allocation.

4.1.2 Features of American Stock Market

The U.S. stock market started from New York Stock Exchange, which was established and began operating in 1811 by agent managers, according to the *buttonwood agreement*. At present the American stock market is mainly composed of the New York Stock Exchange, the NASDAQ Stock Exchange, American Stock Exchange and Over the Count market.

Absolutely, the most important indicators of U.S equity market are consisted of Dow Jones Industrial Average index, Standard & Poor's 500 index and NASDAQ index.

The history of American stock market development can be roughly divided into four historical periods:

1. From the end of the 18th century to 1886, there was a preliminary development in American stock market.
2. In phase of 1886-1886, American stock market developed rapidly, but the market manipulation and the insider trading cases are very serious.
3. The third period of history is between the great depression in 1929 and 1954, and the U.S. stock market experienced an important development under standardization.
4. The last phase is from 1954 until present, institutional investment is developing rapidly, and the U.S. stock market investments come into the modern period.

Besides, the U.S. stock market is the most developed stock market all over the world. Not only stock issuing market and circulation market, but also the number of varieties of stock issuance, trading, and the stock market capacity even the degree of market development, are second to none in the world.

And then there are some basic features of U.S stock market as:

a) Mature, standardized operation, scale, price stability.
b) Strict management, standardized securities market.
c) The United States allow foreign corporations issue stock and traded securities in the United States domestic market.
d) There are plenty of equity exchanges in America.
e) The United States have a developed over-the-counter.
f) There are some important indexes indicating the performance of American stock market.

What is more, the circulation of stock market in the United States is mainly divided into centralized trading market and scattered trading market, the former mainly refers to the Stock Exchange, the latter includes the Over the Counter or OTC market, the third market, fourth market. The circulation market of the United States is the redistribution market of the outstanding shares. In addition, the main task of this market is to provide venues, equipment and professional personnel to promote and facilitate the secondary securities trading.

Last, the importance of U.S stock market cannot be ignored in American economic activities. We can list some main function of it as:

- The stock market is the main source of American enterprise financing.
- The stock market is an important channel to absorb the foreign capital.
- The stock market is crucial leverage of the economic structure adjustment.

4.2 Statistic Analysis Shenzhen Component Index and DJIA Index

In this subchapter, we will pay attention to the stock indexes, which are selected from Chinese stock market and U.S stock market. One index is Shenzhen Component Index compiled by Shenzhen Stock Exchange. Adnother one is DJIA Index, represented by New York Stock Exchange.

Besides this empirical analysis is performed on weekly close price, and the data set is during the 15-year period since January 2000. This sample includes approximately 740 weekly observations.

Why we choose weekly data for our study instead of the daily data, which have more financial high frequency characteristics? Because it is difficult to find the shock effect period depend on daily data. Since there can be a big difference, comparing each day, to find the volatility clustering will be hard and could be wrong. Furthermore, the specifics of shock effect will be described at latter subchapter.

Absolutely, all of the time series data are downloaded from fee-paying stock market trading software, which is created by DZH, a Chinese consultant company.

The proposal of this subchapter is to investigate and introduce the basic information of selected indexes, which include general overview, the index return and statistic features of index logarithmic return.

4.2.1 General Overview of Selected Indexes and Related Stock Exchange

Firstly, let us focus on the Shenzhen Stock Exchange and the Shenzhen Component Index.

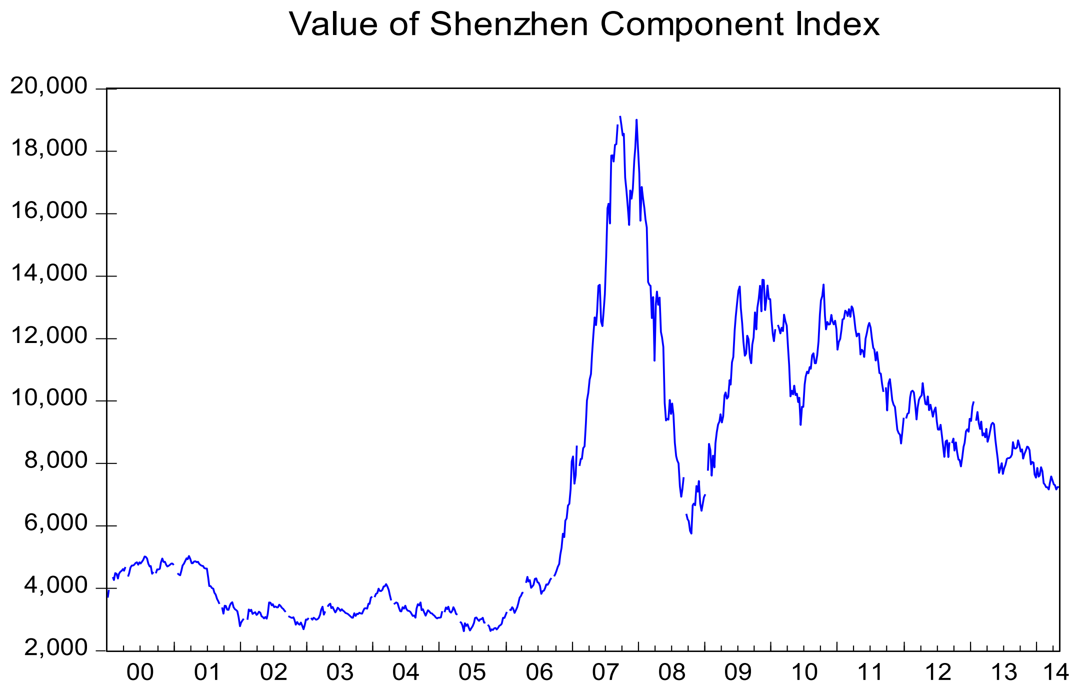
The Shenzhen Stock Exchange or SZSE is the second mutualized national Stock Exchange in China, and established in 12/1/1990. Until now, SZSE includes more than 500 corporations, and there are more than 100 million investors and institutions participate in this stock market. Absolutely, Shenzhen Stock Exchange provides the premises and facilities for concentrated securities trading, organization and supervision under the relevant state laws. At the same time, SZSE is an independent legal person, complied the self-discipline management.

Up to November of 2014, Shenzhen Stock Exchange has achieved 12619.8 billion RMB (approximately 527.6 billion dollars with 6.1936 of exchange rate) of market capitalization. In addition, there are around 600 thousand deals are performed daily on average. Buying and selling orders are matched by automated system under price-time priority.

Shenzhen Component Index is a weighted stock price index compiled by the Shenzhen Stock Exchange. To be more specific, it is constructed by extracting 40 sample shares of object companies from listed stocks in this stock market, which is calculated by weighted outstanding stocks. In addition, the base time is on July 20, 1994, and the base index for 1000 points. Surely, SZSE reflects comprehensively the listed A shares and B shares.

There is a graph of Shenzhen Component Index value, which is created by weekly close price, see Figure 4.1:

Figure 4.1: Weekly data of close price for Shenzhen Component Index



Sources from: Own calculation in EViews, and composition is presented in appendix

In this Figure 4.1 of own calculation, the sample period is from 01/01/2000 to 05/03/2014. According to this line curve, we can roughly divide whole sample period into 3 periods. The first period is between 2000 and 2006, which keep in a stable price level around approximately 4,000 points. In addition, it seems that there is a slight drop in 2001. In the second phase during 2006 ~ 2008, Shenzhen Component Index experience an upward trend just before undergoing a downward trend in a short period. Last period is during 5 years' time since 2008, which witness a decline just after recovery again and again. In this period, price level of index fluctuates around approximately 10,000 points. As a whole, the value of Shenzhen Component Index has a jumping growth trend.

Secondly, another object is the DJIA Index and the related New York Stock Exchange.

The New York Stock Exchange or NYSE, sometimes known as the "Big Board", is the first stock exchange in the United State and established in 1817. Until now, the NYSE is the largest stock exchange market all over the world. Up to February 2015, the market capitalization of NYSE's listed companies was higher than 16.6 trillion dollars. In addition the average daily trading value was about 169 billion dollars.

The trading floor of NYSE is located at 11 Wall Street and is composed of 21 rooms used for the facilitation of trading. The main building, located at 18 Broad Street, between the corners of Wall Street and Exchange Place.

The NYSE is owned by Intercontinental Exchange, an American holding company. Previously, it was part of NYSE Euronext or NYX, which was formed by the NYSE's 2007 merger with the fully electronic stock exchange Euronext. NYSE and Euronext now operate as divisions of Intercontinental Exchange.

A continuous auction format is used in NYSE trading, where traders can execute stock transactions on behalf of investors. They will gather around the appropriate post where a specialist broker, who is employed by a NYSE member firm, acts as an auctioneer in an open outcry auction market environment, to bring buyers and sellers together and to manage the actual auction.

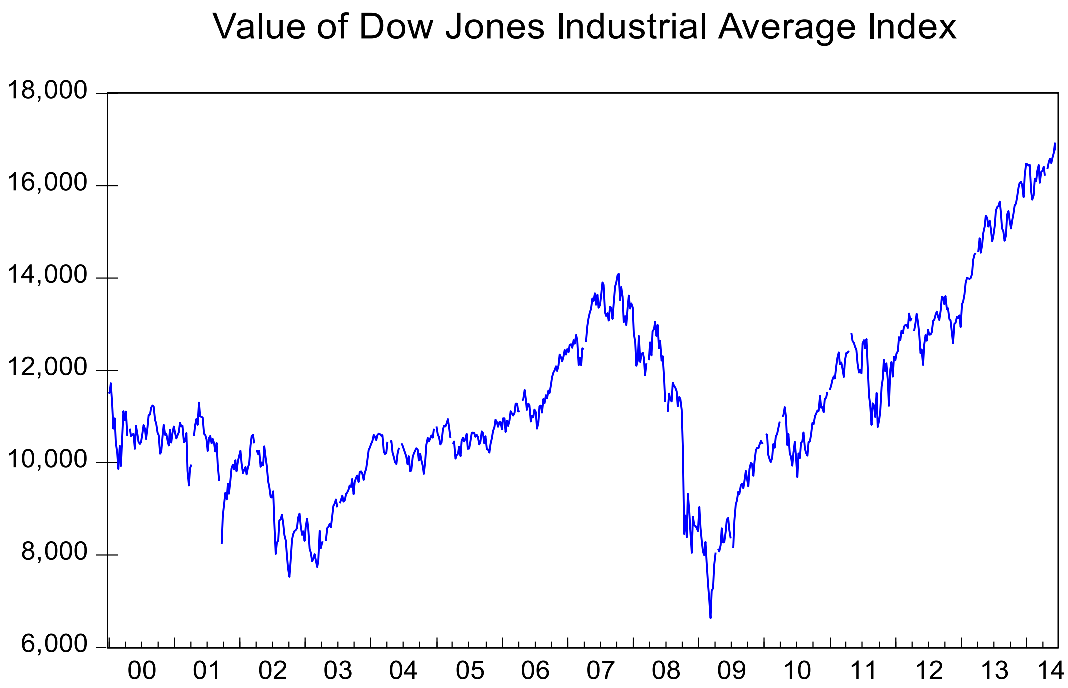
As of January 24, 2007, all NYSE stocks can be traded via its electronic hybrid market except for a small group of very high-priced stocks. Customers can now send orders for immediate electronic execution, instead route orders to the floor for trade in the auction market.

The Dow Jones Industrial Average, also called the Industrial Average, the Dow Jones, the Dow Jones Industrial, the Dow 30, or simply the Dow, is a stock market index in NYSE, and created by Wall Street Journal editor and Dow Jones & Company co-founder Charles Dow.

The Industrial portion of the name is largely historical, as many of the modern 30 components have little or nothing to do with traditional heavy industry. In the past, the calculation method was price-weighted average. In addition, to compensate for the effects of stock splits and other adjustments, it is currently a scaled average. The value of the Dow is not the actual average of the prices of its component stocks, but rather the sum of the component prices divided by a divisor, which changes whenever one of the component stocks has a stock split or stock dividend, so as to generate a consistent value for the index.

Since the divisor is currently less than one, the value of the index is larger than the sum of the component prices. Although the Dow is compiled to measure the performance of the industrial sector within the American economy, the index's performance continues to be affected by many factors. Not only corporate and economic reports, but also domestic and foreign political events will influence its value, such as war and terrorism, as well as by natural disasters that could potentially lead to economic harm.

Figure 4.2: Weekly data of close price for DJIA Index



Sources from: Own calculation in EViews, and composition is presented in appendix

This Figure 4.2 is about DJIA Index value, which is constructed by weekly data. As can be seen from this graph, the sample period is from 01/01/2000 to 06/20/2014. Viewing as a whole, there is a gradual increase trend in DJIA Index, but we can also separate this whole period as 4 phases. In the first period during 2000 ~ 2002, we can see the obvious drop trend. In addition, DJIA Index experiences a sharp rise in the second phase between 2002 and 2007. Besides, a dramatic decline can be seen in the third period from 2007 to 2008. Last, it seem that the DJIA Index undergo a gradually recovery during 2008 ~ 2009.

4.2.2 Indexes Returns and Statistical Analysis

In this part, we will introduce the methodology of returns creation by nominal value at the beginning. And then the figure of weekly logarithmic return will be showed separately. Last there will be some statistical analysis comparison between returns of Shenzhen Component Index and returns of DJIA Index.

As we can see in Figure 4.1 and Figure 4.2, which show that the development of the observed indexes are non-stationary. To develop a good regression model, it would be better, if we use stationary data. There is a general ways of altering non-stationary data into stationary data, where creating a time series of weekly returns and continue using the time series of weekly returns in all calculations showing as followed.

Generally P_t represents the price of an asset in the time t, in our case of analysis, it is the closed price of stocks. Assume that we hold the asset from the time $t-1$ then it will bring the investor a brutto return which is defined as:

$$(1+R_t) = \frac{P_t}{P_{t-1}} \ . \tag{4.1}$$

Netto return can be derived as:

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}} \ . \tag{4.2}$$

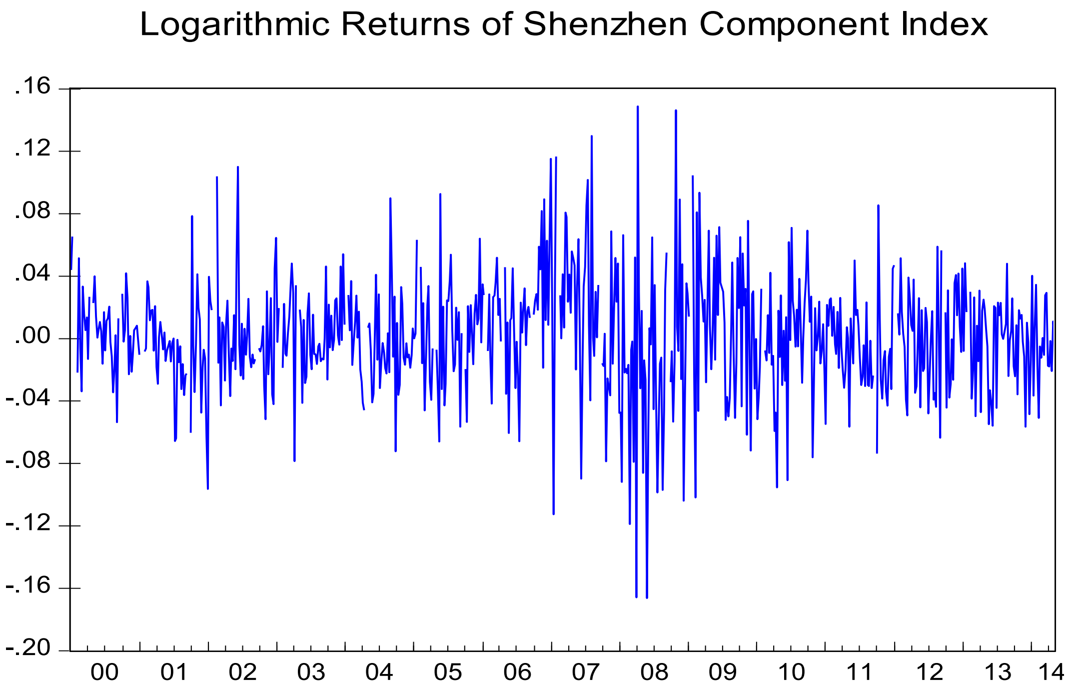
However, this time series of weekly returns do not confirm the assumptions made by normal distribution due to dissociation. So we should change the inference into logarithmic returns for confirming stationary of the weekly return time series with normal distribution as:

$$r_t = ln(1+R_t) = ln\frac{P_t}{p_{t-1}} = ln\,P_t - ln\,P_{t-1} \ , \tag{4.3}$$

where the r_t represents the absolute incremental of logarithmic price and is called logarithmic return.

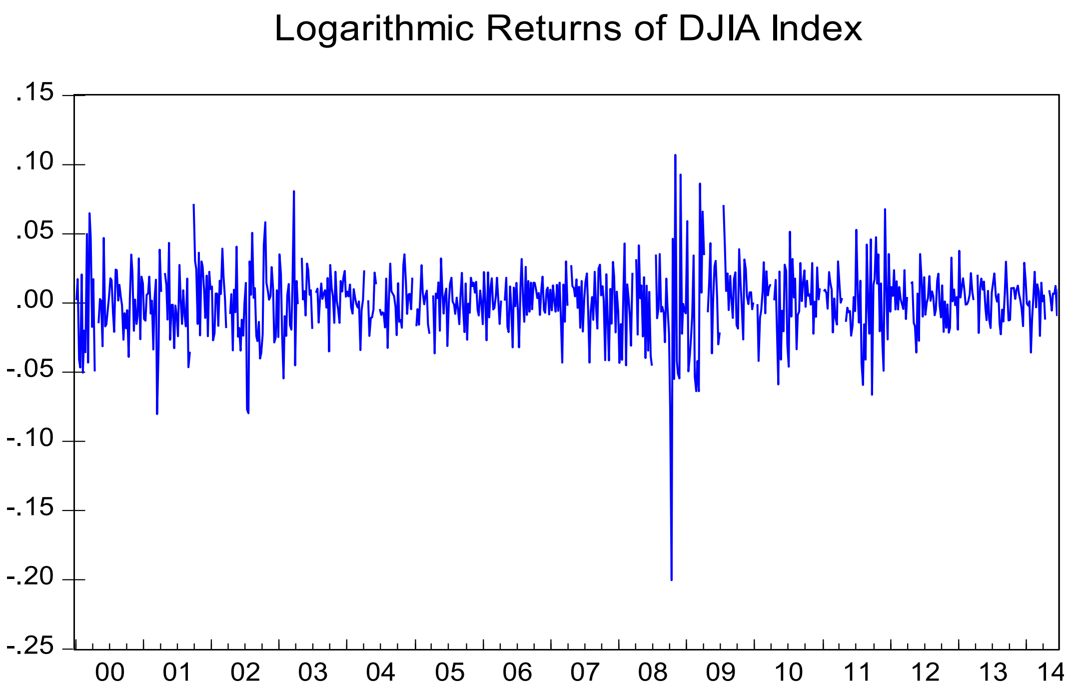
If we apply this formula 4.3 on the index for both selected indexes, we will receive the weekly logarithmic return time series, depended on stock's closed price as Figure 4.3 and 4.4:

Figure 4.3: Weekly logarithmic returns of Shenzhen Component Index



Sources from: Own calculation in EViews

Figure 4.4: Weekly logarithmic returns of DJIA Index



Sources from: Own calculation in EViews

Comparing the Figure 4.3 with Figure 4.4, we can find some difference characteristics between each returns of index. According to our explain in the Chapter 2, it seem that there is more clear volatility clustering in returns of DJIA Index than in returns of Shenzhen Component Index. Furthermore, viewing as a whole, the average volatility of returns in Shenzhen is more than in America, which present the general idea, fluctuation in emerging countries and stability in developed countries. To be more specific, we can find the same biggest volatility in 2008, which is the reaction of global financial crisis.

And then, there is the comparison table, which includes almost all of the crucial statistical features in each selected indexes returns, see as Table 4.1 follow:

Table 4.1: Comparison of descriptive statistics between selected indexes returns

Descriptive Statistics	Shenzhen Component Index	DJIA Index
Mean	0.14%	0.06%
Maximum	14.89%	10.70%
Minimum	-16.88%	-20.03%
St.deviation	0.0382	0.0242
Skewness	-0.0940	-1.0510
Kurtosis	5.1543	12.2395
J-B test	145.9413	2828.2890
Probability	0.0000	0.0000

Sources from: Own calculation in EViews

When we pay attention to this Table 4.1, it is apparent that the means of both sample returns are relatively small, which is close to 0%. However, both standard deviations are significant higher than their average returns. To be more specific, both series of weekly index returns show mostly leptokurtic distribution, which has a higher peak and heavy tail, as we mentioned in Chapter 2. There are 3 evidences of leptokurtic distribution, negative skewness or left-skewed, the big value of kurtosis compared with 3 and the significant high value of Jarque-Bera test (normality test mentioned in Chapter 2). Thus the hypothesis of normality can be rejected in both markets.

Comparing the descriptive statistics between selected indexes returns in this 15-year period, it can be seen that almost all of the statistical features are bigger in Shenzhen Component

Index than DJIA index, except kurtosis and the J-B test. So we can conclude the volatility is higher in Shenzhen on average than in America. Furthermore, as we described in Chapter 2, it is seems that there is also a leverage effect, presented by the asymmetry between positive returns or maximum and negative returns or minimum.

4.3 Identification of Sudden Breaks in Volatility

In order to identify the number of sudden breaks in volatility, and get each time period of return series, we used the ICSS algorithm for determining each phase and calculating the standard deviations between the shock points. There is a Table 4.2, which indicates the time periods of sudden breaks in volatility as described by the ICSS algorithm in Chapter 3. In addition, the most important events are defined and chosen in this Table 4.2, as followed:

Table 4.2: Sudden breaks in volatility as detected by modified ICSS algorithm

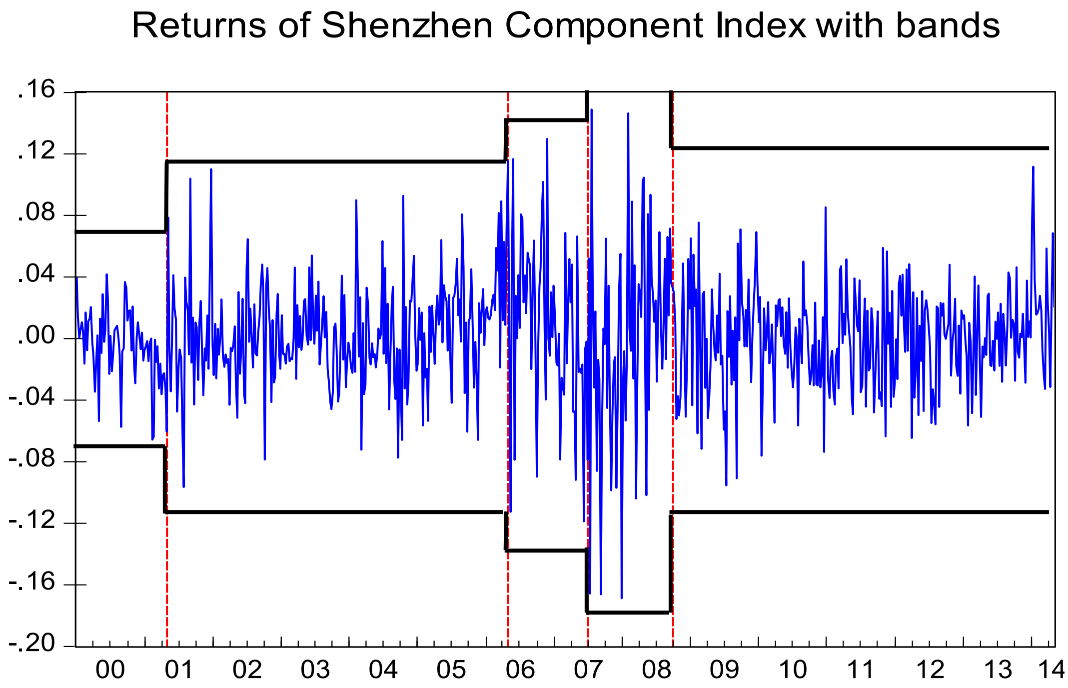
Index	St. Deviation	Time Period	Economic Event
Shenzhen Component Index	2.14	01/2000 – 05/2001	Before entering WTO, Relative closed economy, Lack regulation in stock market
	3.75	05/2001 – 05/2006	China entering WTO, Economic growth rapidly, Development in real estate industrial, Strengthen regurlation
	4.68	05/2006 – 07/2007	Transformation of exchange rate regime, Hot money inflow, Expansionary monetary and fisical policy, Real estate bubbles
	5.82	07/2007 – 10/2008	Global financial crisis
	3.79	10/2008 – 06/2014	Economic growth, Period of relative stability, Tansform in economic structure
DJIA Index	2.76	01/2000 – 11/2000	Economic growth, Period of stability
	5.17	11/2000 – 06/2001	Dot com bubbles
	3.38	06/2001 – 11/2002	Recovery of markets
	1.79	11/2002 – 03/2008	Economic growth
	6.98	03/2008 – 01/2009	Global financial crisis, US sub–prime mortgage crisis
	2.87	01/2009 – 06/2011	U.S. credit rating decrease, Fears of a possible return of the global economy into recession, Investors to move away from more risky assets to safer.
	2.03	06/2011 - 06/2014	Economic growth, Period of relative stability

Sources from: Own analysis and calculation in R (a software), and the calculation results in R is presented in appendix

Where the value of standard deviations are presented as some times of standard deviation or σ, and the value is the up and down limitation of volatility for each given regime.

According to the presentation of volatility periods in Table 4.2, we can create the index returns with bands, as Figure 4.5 and Figure 4.6.

Figure 4.5: Volatility with bands in Shenzhen market



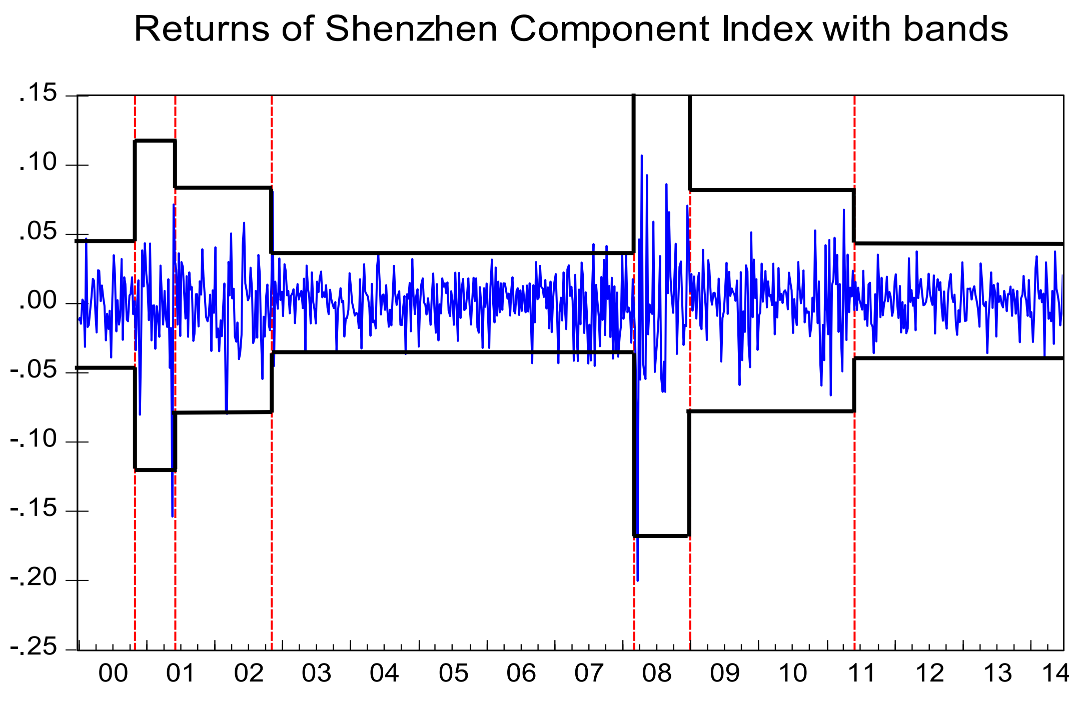
Sources from: Own calculation in EViews

Focused on the Figure 4.5, it seems that the asymmetric volatility is not significant in returns of Shenzhen Component Index. Because with ± 3 standard deviations bands calculated for the given regime, it is almost include all of the observation in each regime.

To be more specific, the volatility has a significant jumping trend before financial crisis. The first dominant increase in volatility occurred right after the event that China entered into World Trade Organization or WTO in December 2001. Before entering WTO, China was a relative closed economy, and it was difficult to enter Chinese capital market even it is still inefficient. However, there were a lot of hot money inflow and a great GDP development, when China took a part into WTO. And then the second volatility jumping was in May 2006, which was caused by the influence of exchange rate transformation around July 2005. After transformation of exchange rate, the inflow of hot money was accelerated, and the price of real estate touched the peak. There was a huge bubbles in Chinese economy, and a higher level of volatility during 2006 ~ 2007 than previous period. It can be observed that bubbles broken, when the global financial crisis was coming from 07/2007 to 10/2008. During this crisis, the volatility jumped into the highest level, comparing with other periods in the selected sample.

It is easy to find that the volatility of returns turned back to a relative stability degree after global financial crisis. Although the transformation of economic structure has been pushed, the bubbles were still big, especially in real estate industry. In this period, there was still a huge risk of uncertainty in Chinese economy.

Figure 4.6: Volatility with bands in American market



Sources from: Own calculation in EViews

As we can see in Figure 4.6, there are lots of significant different features of volatility in DJIA Index returns, comparing with the volatility of Shenzhen. It is easy to see that some of negative returns are out of the bands for the given regime, with ±3 standard deviations bands. So the volatility is much more followed the rule of asymmetric volatility in American stock market.

Widely, American stock market is regarded as the representative of developed stock market. There are some typical shock breaks, which are reflected in the volatility of index returns, like as in our sample time series:

- The IT dot com bubbles of 2000.
- The 9/11 terror attack in 2001.
- The recent financial crisis during 2007 ~ 2009 years.

In addition, there is generally a recovery in market during the period from a crisis to another crisis. It seems that the volatility is significant smaller in recovery term than other periods.

Overall, the volatility influence of global event in one financial market can be transmitted to other market, as the global financial crisis or sub-prime mortgage crisis. Nevertheless, local event could create some significant effect, especially in emerging countries as China. Based on the variance of returns, we can conclude that the higher risky market is in China rather than the U.S.

4.4 Estimation of Conditional Volatility Models

After the identified sudden changes according to volatility in last Subchapter 4.3, we will discuss conditional volatility estimation models in this subchapter. As we mentioned in Chapter 3, the GARCH(1,1) model and FIGARCH(1,d,1) model will be presented, with and without dummy variables. Absolutely, to investigate the influence of sudden shifts on volatility persistence or long memory is the second step of goal in this thesis.

Besides, the distribution of both ARMA model follow the Generalized Errors Distribution or GED. Because the most situation of distribution is not normality according to practical experience, the most popular adjustment is general errors distribution.

Table 4.3 and 4.4 show the results of the GARCH(1,1) and FIGARCH(1,d,1) models with and without dummy variables, as we can see:

Table 4.3: Parameters of GARCH (1,1) model with and without dummy variables

GARCH(1,1) model without dummy variables						
Index	α_1	β_1	*Volatility Persistence*		*Q(10)*	*AIC*
Shenzhen	0.056*	0.930*	0.986		22.950*	-3.852
DJIA	0.178*	0.760*	0.938		8.264	-4.897
GARCH(1,1) model with dummy variables						
Index	α_1	β_1	*Volatility Persistence*	*Persistence Decline*	*Q(10)*	*AIC*
Shenzhen	0.032*	0.606*	0.638	0.348	26.819*	-3.682
DJIA	0.105*	0.569*	0.674	0.264	12.619	-4.281

Sources from: Own calculation in EViews

Where the * denotes that correspondent estimated value is statistically significant at 5% significance level. In addition, the Ljung–Box statistics Q_n are used for test whether the null hypothesis of no autocorrelation should be rejected in the n-lag squared return series, which is similar as variance or σ^2. According to the Formula 3.12 mentioned in Chapter 3, the * denotes that it is fail to reject the alternative hypothesis of autocorrelation for this white noise or u_t. As for the AIC, it is the Akaike information criterion described in Chapter 3. We can choose the best suitable model by comparison of AIC value.

In our thesis, the autocorrelation of white noise, u_t, have not make crucial influence for model estimation. So we ignore the effect of the result that hypothesis of no autocorrelation should be reject in modeling of Shenzhen. According to practical experience, volatility model is dominant in estimation, and the autocorrelation is not important.

According to the GARCH(1,1) model without dummies exhibition, there are 2 highly significant parameters, α_1 and β_1, at 5% significance level. Moreover, the sums of these 2 parameters are almost close to 1 in total models. In other words, the sudden shifts have a persistence impact or long memory on the volatility whole investigated time series. The linear volatility estimation model will be worse with a higher long memory, which means there will be a long persistence influence after a sudden shock event.

How to decrease the effect of long memory? We can test the linear volatility estimation model with some dummy variables at each sudden break point, as seen Table 4.3. So it is clear that there is a significantly decline effect in long memory after adding some dummy variables. Shenzhen Component Index shows a drop in volatility persistence of 0.348, while DJIA Index presents a smaller decrease in volatility persistence of 0.264. Based on this result, it seems indicate that the standard GARCH(1,1) model tends to overestimate volatility long memory, when ignoring sudden shifts in conditional volatility. Moreover, if compare both indexes with and without dummy variables, we can find the situation that Shenzhen Component Index have longer volatility persistence but lower after decline, than DJIA Index. It seems mean that there is a longer influence after sudden break in Shenzhen than in America.

Last, we should choose the best suitable GARCH(1,1) model by Akaike information criterion. Comparing the value of AIC in Table 4.3, for both indexes there is a bigger negative value before adjusting by dummy variables. So the GARCH(1,1) model without dummy variables is much better for both Index. However, a selection of proper models will be presented in next Subchapter 4.5, according to in-sample forecasting by loss function.

Table 4.4: Parameters of FIGARCH (1,*d*,1) model with and without dummy variables

FIGARCH(1,*d*,1) model without dummy variables					
Index	α_1	β_1	*d*	*Q(10)*	*AIC*
Shenzhen	0.053*	0.907*	0.686*	23.987*	-3.897
DJIA	0.167*	0.729*	0.482*	8.723	-4.921
FIGARCH(1,*d*,1) model with dummy variables					
Index	α_1	β_1	*d*	*Q(10)*	*AIC*
Shenzhen	0.069	0.639	0.094	25.963*	-3.752
DJIA	0.192*	0.629*	0.261	13.63	-4.825

Sources from: Own calculation in R (a software)

Focused on the Table 4.4 in the FIGARCH(1,*d*,1) models without dummy variables, we can find that the difference parameter *d* is statistic significantly from those cases, when

$d = 0$ (FIGARCH model is same as GARCH model) and $d = 1$ (FIGARCH model is same as IGARCH model). As we described in Chapter 3, d is used for measuring the effect of long memory, and the higher value of d the longer memory effect will be.

According to the presentation of Table 4.4, it seems that volatility of both series reveals a persistence impact by statistic significant criterion. Nevertheless, if including sudden breaks into the FIGARCH(1,d,1) model by adding some dummy variables, the values of estimated parameters d is reduced significantly. Although the volatility persistence disappears after adding the dummy variables into FIGARCH model, the model cannot be chosen due to the statistical insignificance of d at 5% level.

Last, if we compare the persistent parameters between d and $\alpha_1 + \beta_1$, it can be seen that both volatility persistence in FIGARCH(1,d,1) are lower than in GARCH(1,1) without dummy variables, even in GARCH model of DJIA Index with dummies. In addition, when we compare the Akaike information criterion between GARCH(1,1) and FIGARCH(1,d,1), we can make the conclusion that the FIGARCH(1,d,1) model without dummy variables is most suitable for volatility forecasting of both Index.

To sum up, if we can control the structural sudden breaks in conditional variances, volatility persistence will be reduced sufficiently. As a result in all stock market around world, the long memory situation in volatility is usually exaggerated by shock breaks corresponding, especially in global financial events.

Finally, a good regression model has to fulfill certain conditions such as constant residuals, no autocorrelation and the model should not have normal distribution. There is a statistic test, called diagnostic test, which is used to test whether the estimated models have met the required conditions. According to the results of residual diagnostic as provided for indexes return series, all of models suggested that the hypothesis of no autocorrelation and homoscedasticity cannot be rejected. In addition, the probability distributions of residual series are not normal as provided by J-B test, so all of models were explained as followed Generalized Errors Distribution (GED) instead of normal distribution.

4.5 In-sample Forecast of GARCH(1,1) Model

After estimation modeling of conditional volatility in last subchapter, we will test the quality of GARCH(1,1) model with and without dummy variables by loss functions. As we mentioned in Chapter 3, the lower is value of loss functions, the higher is the predictive ability of the estimated model. And then evaluate the explanation effect of model, by comparing the fitting degree in figure. Why we skipped FIGARCH(1,*d*,1) model? Because some parameters in FIGARCH(1,*d*,1) model with dummy variables were not statistic significant, it made no sense to compare the quality between the model with dummy variables and without dummy variables.

There are 3 most popular loss functions for quality test by difference comparison between actual data and estimated data, such as RMSE, MAE and Theil's Coefficient. In addition, the forecasting is about in-sample forecast about the historical data rather than out-of-sample forecast about future estimation.

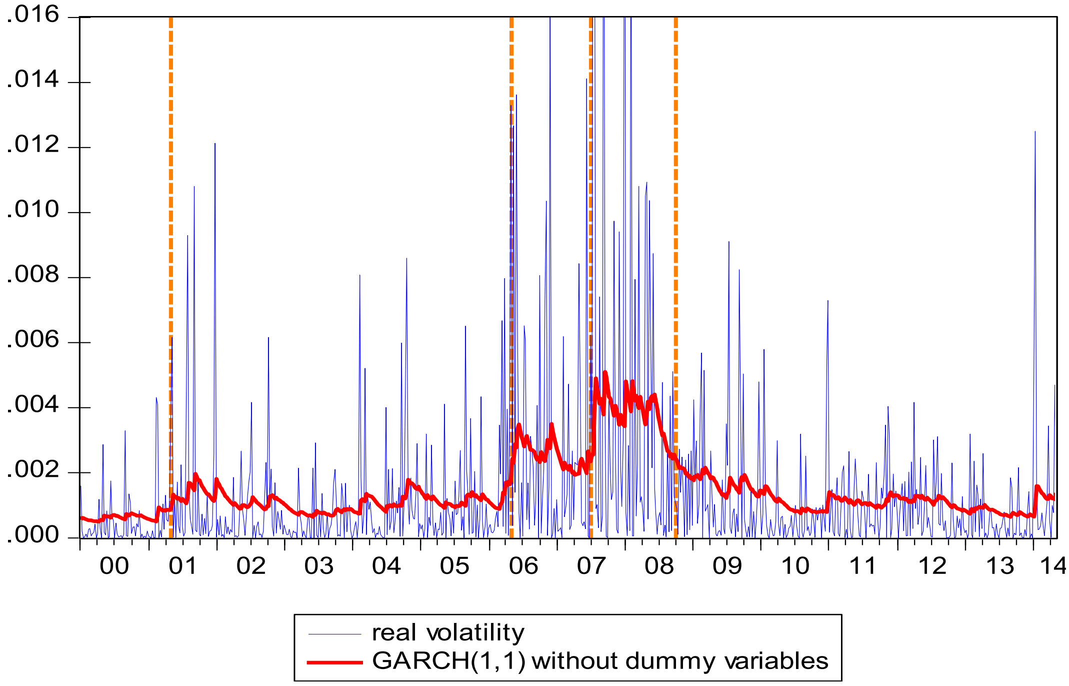
Firstly, we will analyze the estimation model for Shenzhen Component Index as demonstrated by Table 4.6:

Table 4.6: Results of loss functions for Shenzhen Component Index

Shenzhen Component Index	RMSE	MAE	Theil´s Coefficient
GARCH(1.1) without dummy	0.00283005	0.00154143	1.014108
GARCH(1.1) with dummy	0.00224248	0.00113633	0.816089
Change after dummy adding	-20.76%	-26.28%	-19.53%

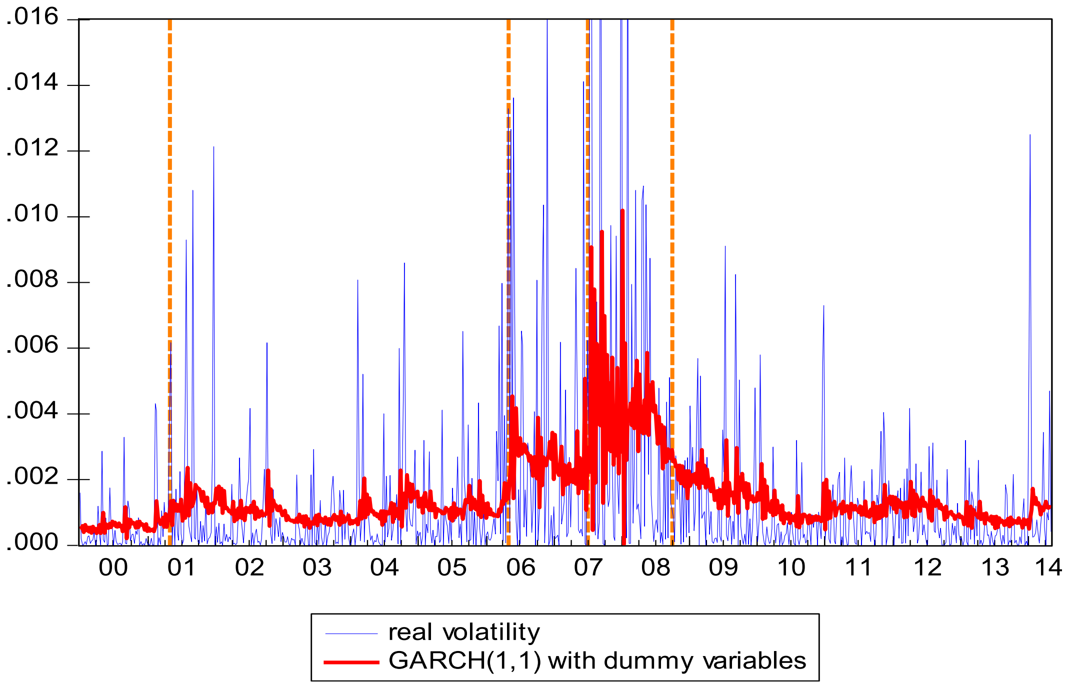
Sources from: Own calculation in EViews

Figure 4.7: Conditional variance of Shenzhen without dummy variables



Sources from: Own calculation in EViews

Figure 4.8: Conditional variance of Shenzhen with dummy variables



Sources from: Own calculation in EViews

Where the real volatility is defined as:

$$real\ volatility_t = \frac{\sum_{t=1}^{1}(returns_t - 0)^2}{1} = returns^2 \ . \tag{4.1}$$

According to the description in Table 4.6, it can be seen that values with dummy variables are smaller than without dummy variables in all results of loss function. Besides, the decline after adding dummy variables is 21% in RMSE, 26% in MAE and 20% in Theil's Coefficient.

Furthermore, if we compare the estimation conditional variance without dummy variables in Figure 4.7 and with dummy variables in Figure 4.8, the result of Table 4.6 is intensified. And then it is easy to see that the conditional variance is explained more significantly in Figure 4.8 rather than Figure 4.7. In addition, we can find the highest conditional variance is happen during global financial crisis. And the smallest conditional variance period is before May 2001.

To sum up, we can create the conclusion that the better volatility estimation model is GARCH(1,1) with dummy variables instead of model without dummy variable for returns of Shenzhen Component Index. However, if we compare the result of Akaike information criterion, in previous Subchapter 4.4, with loss functions, the conclusions are contrary about the selection of model.

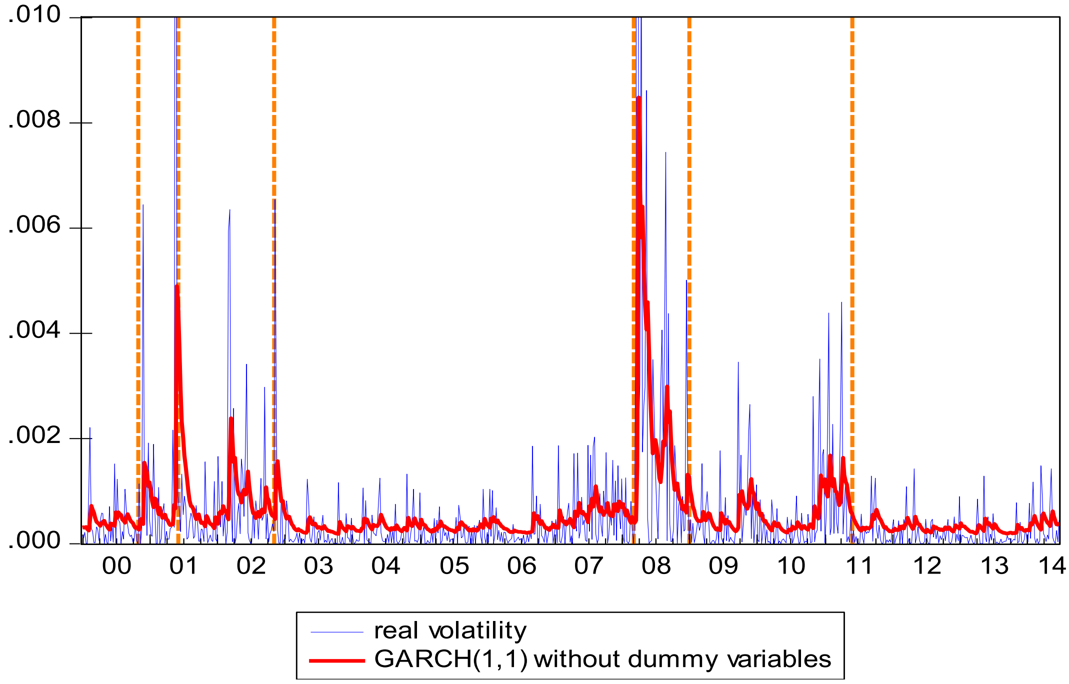
Secondly, there will be the analysis of estimation conditional volatility in DJIA Index as demonstrated by Table 4.7:

Table 4.7: Results of loss functions for DJIA Index

DJIA Index	RMSE	MAE	Theil´s Coefficient
GARCH(1.1) without dummy	0.00189646	0.00061440	0.57500775
GARCH(1.1) with dummy	0.00130312	0.00043286	0.37753647
Change after dummy adding	-31.29%	-29.55%	-34.34%

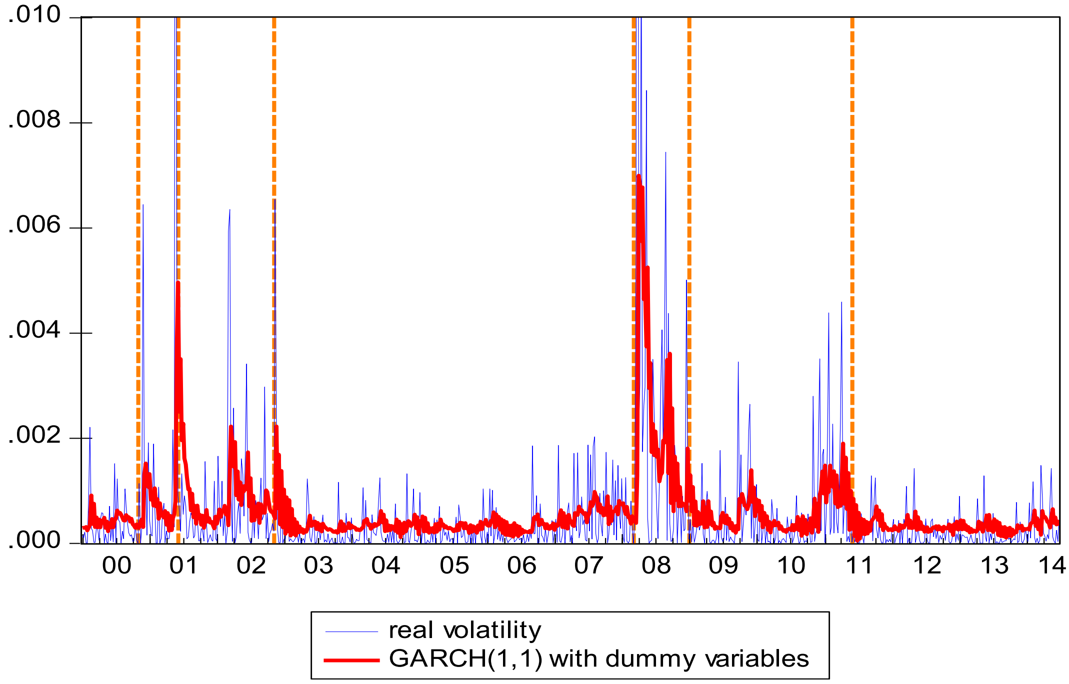
Sources from: Own calculation in EViews

Figure 4.9: Conditional variance of DJIA without dummy variables



Sources from: Own calculation in EViews

Figure 4.10: Conditional variance of DJIA with dummy variables



Sources from: Own calculation in EViews

Based on the quantitative data presented in Table 4.7, we can find the similar situation that value of loss function is lower after adding dummy variables. And the reduce effect of dummy variables is 31% in RMSE, 30% in MAE and 34% in Theil's Coefficient.

In contrast, it is difficult to find a significant difference of fitting degree between Figure 4.9 and Figure 4.10, when we compare the estimation conditional variance without and with dummy variables. So it can just say that there is little improving after inserting dummy variables, as our observation. Absolutely, it can be also observed that the highest conditional variance is happen during global financial crisis. In addition, the second highest volatility period is during Dot Com bubble.

Overall, we can also conclude that the quality of GARCH(1,1) with dummy variables is the better conditional variance model for DJIA Index returns, than GARCH without dummy variables. But the conclusion is also opposite by comparison with analysis of Akaike information criterion.

Finally, it must be realized that the same conclusion as in-sample forecast may be not received in real forecasting evaluation by out-of-sample forecast.

5. Summary

In this thesis, the main goal was investigating the influence of structural breaks caused by exogenous factors on volatility, by using the ICSS algorithm. Since Chapter 4 was the application part, which combined the methodology mentioned and the empirical financial data, all of our goals were analyzed and solved in this part. In this Chapter 5, it is necessary to make the summary for Chapter 4.

First of all, there was the introduction of investigated stock market, one is Chinese stock market, and another one was the American market. As we mentioned in the Subchapter 4.1, understanding about our selected stock market is significant for identifying the economic shock point for each period. Absolutely, it is worth for studying about Chinese and American stock market, because both of them take a significant part in global economy, especially in stock market. In addition, it makes a sense in comparing the difference of performance between emerging country and developed country.

To sum up, Chinese stock market developed rapidly till now, but there are also some unfair problem and inefficient allocation in this market. Besides, American stock market share the most dominant proportion in global stock trading, and make a wide influence when something happen in this market.

In Subchapter 4.2, the statistical analysis of selected index was presented, which is an important research of this thesis. At first, we must determine which exchange and relative index should be investigated in each stock market. And then, the weekly close prices of indexes during 15 years since January 2000, were chosen as the sample for empirical application. Afterwards, we showed the general overview for Shenzhen Component Index and DJIA Index, included the roughly analysis of their value trend. Last, there is the statistical analysis of some parameters for the logarithmic returns of both indexes.

According to the result of descriptive statistics table, especially in the probability of normal distribution hypothesis by Jarque-Bera test, we can conclude that both returns of indexes are not normal distribution, and it seems that there is a bigger volatility in Shenzhen Component

Index than DJIA Index. What is more, the volatility clustering was more significant in American market than Chinese, by observation of Figure 4.3 and 4.4.

Turn to the main goal, the sudden breaks in volatility of both index were identified in the Subchapter 4.3, as the first step of this thesis. Initially, we received the numbers of sudden breaks in volatility and created the Table 4.2 for the specifics of each period, by using the method of ICSS algorithm mentioned in Chapter 3. In addition, items of Table 4.2 were consisted of standard deviation, time period and some important event.

After that, we constructed 2 new figures about logarithmic returns of indexes, which included some volatility bands. In addition, those bands is made as the ± 3 times standard deviations in each given periods, according to the results of ICSS algorithm. And then, there would be some specific analysis by observing the Figure 4.5 and Figure 4.6. Based on the presentation of both figure, we can found that the asymmetric volatility is dominant in American stock market than Chinese.

Overall, it can be concluded that there were identified different economic or political events in both markets, which caused some sudden breaks in volatility. In addition, with reference to the shock events analysis in each stock market, another conclusion can be created as that the volatility influence of significant global event is across different financial market. However, developing countries' stock markets are not mature, and these markets would be shocked easily by local important events. For example, the global events include Dot Com bubbles and global financial crisis, and the special local events like transformation of exchange rate regime in Chinese market and credit rating of sovereign debts decline in American market.

Moving on to the Subchapter 4.4, there was the important step to solve the main goal, which was the impact determination of sudden breaks on volatility persistence, by analysis of conditional variance modeling. First and foremost, GARCH(1,1) and FIGARCH(1,d,1) model were be constructed with and without dummy variables. In addition, adding dummy variables was a significant method to reduce the long memory effect, which was identified by the ICSS algorithm in Chapter 3 and was proved as we seen in Table 4.3 and Table 4.4 in

Chapter 4. Subsequently, we found that the parameters of FIGARCH(1,*d*,1) model with dummy variables were statistic insignificantly at 5% significant level. Thus, FIGARCH(1,*d*,1) model with dummy variables would not be discussed in this thesis.

Following this, when we paid attention to the volatility persistence, it seems that the conclusion, FIGARCH(1,*d*,1) model is better than GARCH(1,1) model for both volatility of index returns, could be received especially with dummy variables. Nevertheless by contrast, we preferred to use GARCH(1,1) model with dummy variables instead of FIGARCH(1,*d*,1) model without dummy variables, in returns series of Shenzhen Component Index. Afterwards, according to Akaike information criterion, it was difficult to reject that FIGARCH(1,*d*,1) model without dummy variables had a better fitting degree than, not only GARCH(1,1) model without dummy variables but also model have dummies.

As a whole, if we can control the structural sudden breaks in conditional volatility, like adding dummy variables into conditional variance model, the volatility persistence will be reduced sufficiently. Based on this conclusion, it seems that the effect of long memory is often exaggerated when shock breaks happening, like the period of global financial crisis in 2007 ~ 2009 years.

In the final stage, the loss functions would be used for testing the quality of forecasting power in estimation model. However, according to the result of parameters significant test in previous Subchapter 4.4, we would not consider the quality comparison between FIGARCH(1,d,1) model without and with dummy variables. Surely, comparing the explanation degree in actual situation and selecting the best one, are also the last step for our goal in this thesis. In addition, it must be claimed that the forecast is in-sample rather than out-of-sample.

Firstly, the results of loss functions were presented, which included the value of RMSE, MAE and Theil's Coefficient. According to the item of change after dummy adding, we could find the significant decline in loss function for both markets, as we can see in Table 4.6 and Table 4.7. And it means the GARCH(1,1) model with dummy variables would be better than no dummy, which was an opposite claim comparing with the result of AIC in Subchapter 4.4.

Furthermore, the comparison of figures enhanced our conclusion that adding dummy variables can increase modeling quality for forecasting. Thus, incorporating information on sudden changes in conditional variance may improve the accuracy of estimating volatility dynamics.

To sum up, there is a summary of proper model selection, which is presented as follow Table 5.2:

Table 5.2: Selection of Estimated Conditional Variance Model

Models \ Criteria		Volatility Persistence	AIC	Loss Functions		
				RMSE	MAE	Theil′s Coefficient
Shenzhen Component Index	GARCH(1, 1) without dummy					
	GARCH(1, 1) with dummy	*		*	*	*
	FIGARCH(1, d, 1) without dummy		*	/	/	/
DJIA Index	GARCH(1, 1) without dummy					
	GARCH(1, 1) with dummy			*	*	*
	FIGARCH(1, d, 1) without dummy	*	*	/	/	/

Where * means this model is the best suitable one in related criterion and index. In addition, / denotes this model is not discussed under special criterion.

6. Conclusions

As we mentioned in the previous chapters, the main aim of this thesis was to identify sudden changes and its influence on volatility persistence, for developed American market and emerging Chinese market by modified ICSS algorithm. And then, it was important to understand the GARCH (1,1) and FIGARCH (1,d,1) models, which was used for long memory analysis on volatility in our thesis. Furthermore, in-sample forecasting took also an important role in quality test of GARCH (1,1) model.

The whole thesis can be divided into 6 parts totally, including the first part: introduction and the last part: conclusion.

In Chapter 2, we gave the descriptions of some features and application about high frequency financial data. Those contents included characteristics of financial time series, assumptions of financial time series and microstructure of financial market.

In Chapter 3, the methodologies of volatility model were introduced, included ARCH, GARCH and FIGARCH models. Besides, the criteria of model selection and ICSS algorithm were identified in this chapter.

Chapter 4 was the part of empirical application in this thesis. In this chapter, the main goal was completed. Not only identification of shock breaks and analysis of volatility persistence, but also in-sample analysis for modeling quality could be seen in this chapter.

After this, we listed and describe all of results from Chapter 4 in Chapter 5.

According to our empirical application in Chapter 4, we can conclude that the volatility influence of significant global event is across different financial market, like the global financial crisis. Nevertheless, emerging countries' stock markets are not mature, and these markets would be shocked easily by local important event.

What is more important, if we can control the structural sudden breaks in conditional volatility, like adding dummy variables into conditional variance model, it can be found that the volatility persistence reduce sufficiently. Based on this conclusion, it seems that the effect of long memory is often exaggerated when major economic and political events happening, like the period of global financial crisis in 2007 ~ 2009 years.

In the case of shock breaks, stock price on financial markets usually do not reflect real prices of underlying asset. The excessive level of volatility can lead to inefficient resource allocation and upward pressure on interest rate because of an excessive uncertainty in equity market.

As we mentioned in Chapter 5, in emerging country especially China, it seems that the influence of excessive volatility is serious. In addition, it is difficult to find out the relationship between the stock price and economic growth in China, due to not only inefficient resource allocation but also some intervention by stakeholders. Furthermore, the excessive volatility may bring some potential errors in risk management, like analysis of Value-at-Risk, option pricing and so on. Therefore, if a volatility model can reflect the impact of new information on markets before harm to economy, this model will reduce the uncertainty risk and be desirable.

Eventually, although all of goals in this thesis were fulfilled, some arguments still existed. For example, the choice of better suitable model, between GARCH (1,1) or FIGARCH (1,d,1), can be different, when different criteria are used for fitting degree test. In other ward, if these arguments are solved, there will be some improvement in this thesis.

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List of Abbreviations

OLS	Ordinary Least Squares
JB	Jarque-Bera
Cov	Covariance
Corr	Correlation
Std.dev	Standard Deviation
ARCH	Autoregressive Conditional Heteroskedasticity
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
ARMA	Autoregressive Moving Average
FIGARCH	Fractional Integrated Generalized Autoregressive Conditional Heteroskedasticity
AIC	Akaike information criterion
SBC	Schwartz-Bayes Criteria
RMSE	Root mean square error
MAE	Mean Absolute Errors
MAPE	Mean Absolute Percentage Errors
ICSS	Iterative Cumulative Sum of Squares
SZSE	Shenzhen Stock Exchange
NYSE	New York Stock Exchange
DJIA	Dow Jones Industrial Average

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ZHUO ZHANG

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